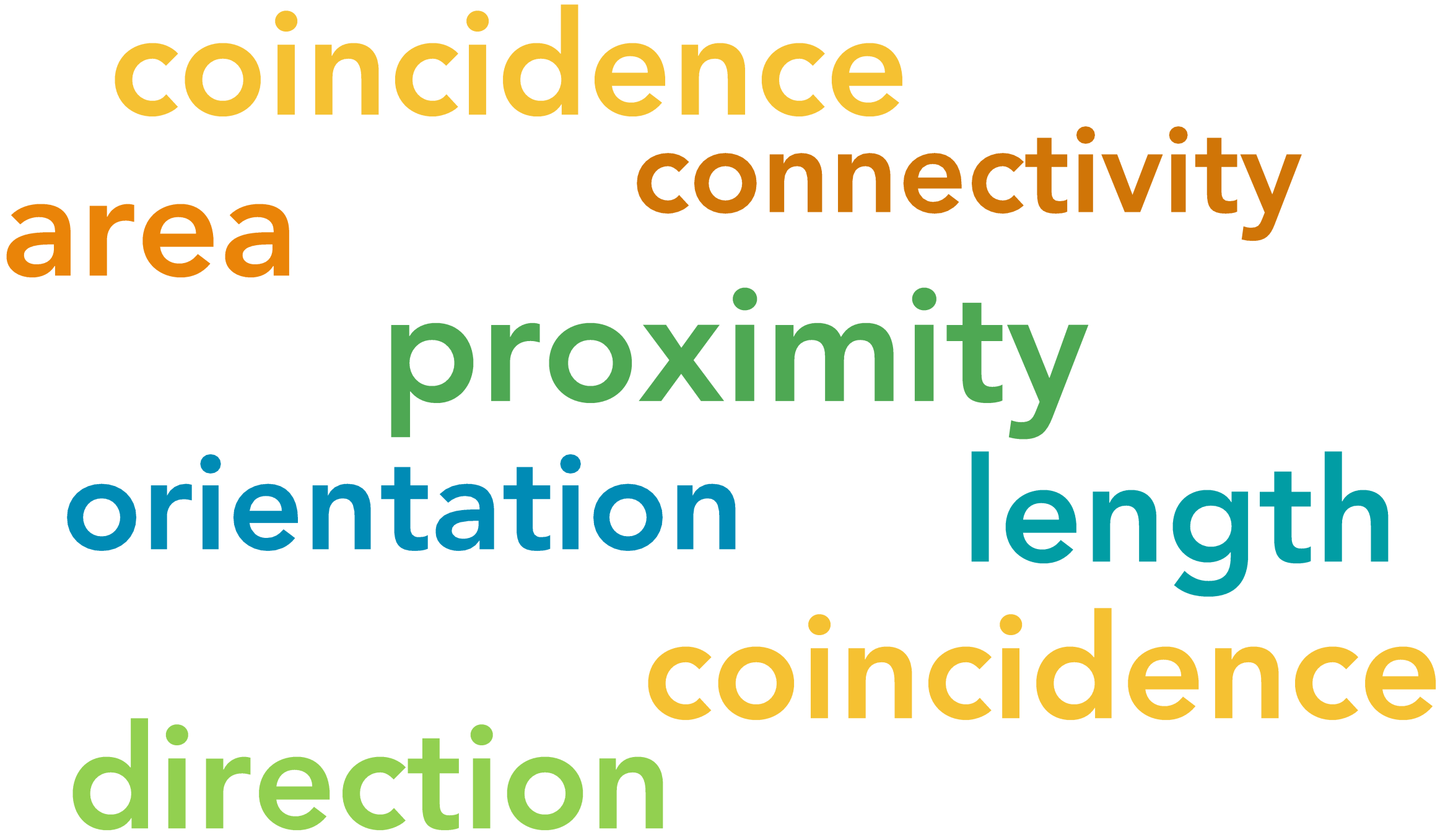
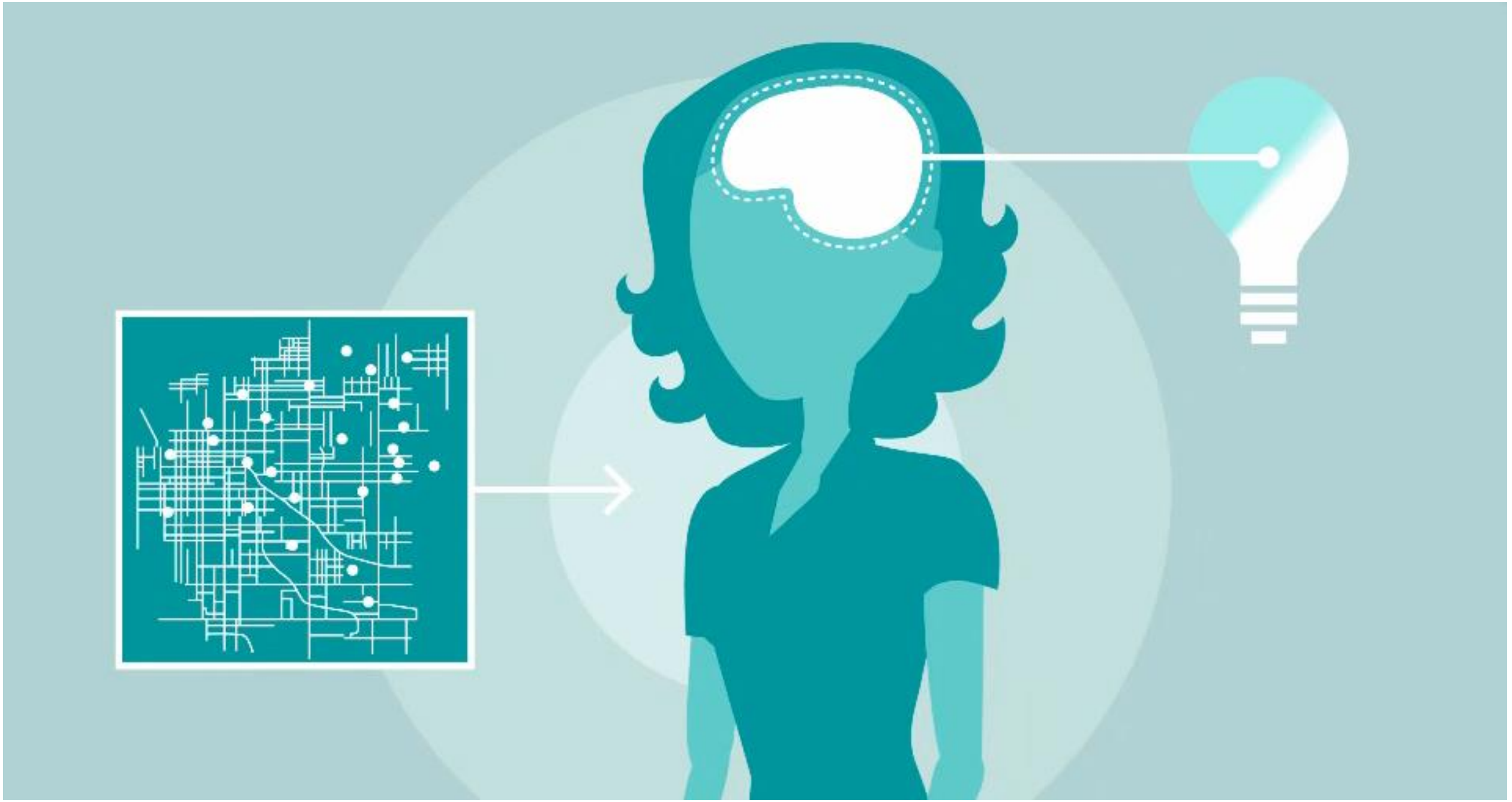
Spatial Statistics: Simple Ways to Do More with Your Data

Lauren Bennett
Flora Vale
Jenora D'Acosta

What are Spatial Statistics?

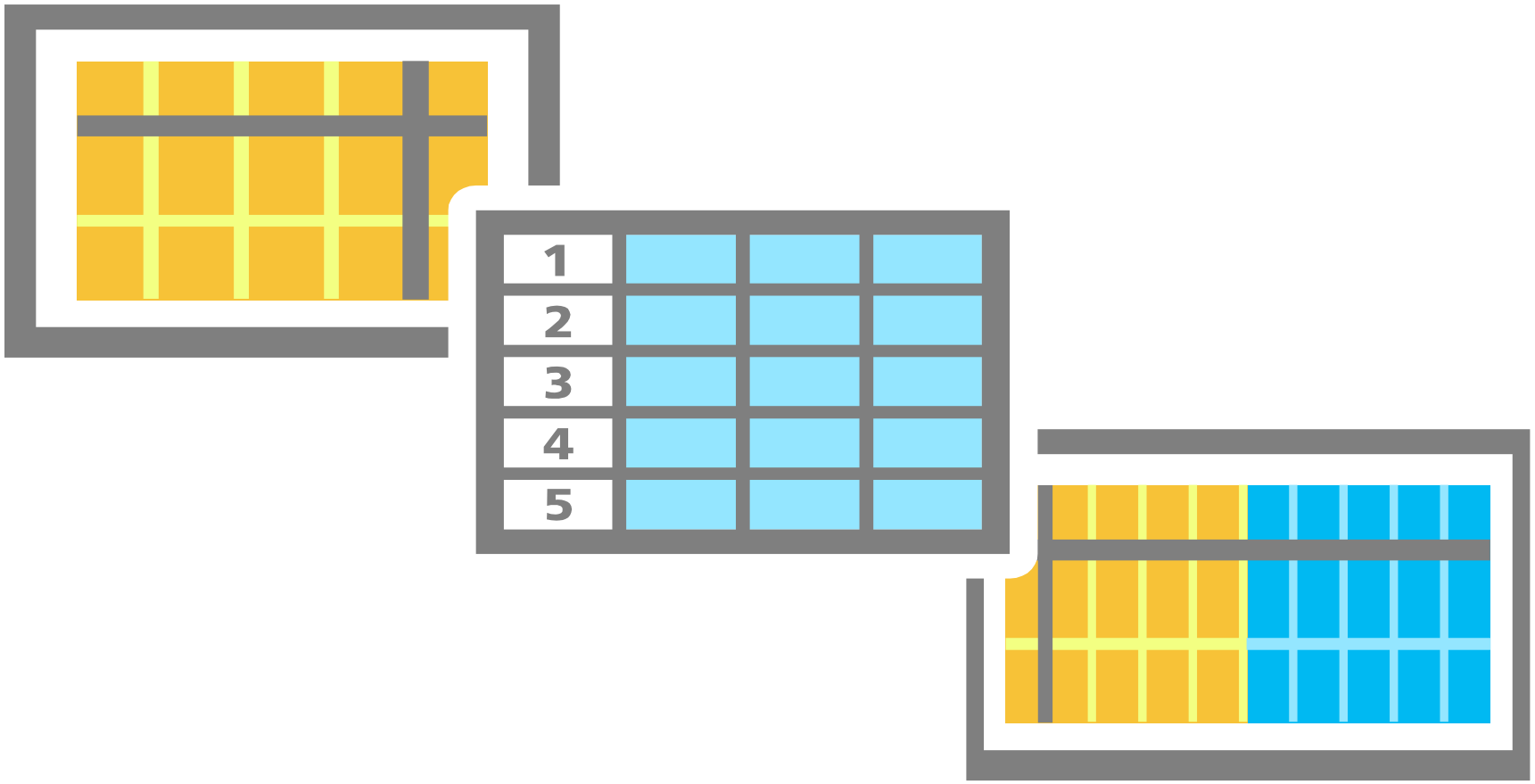
Spatial Statistics are a set of exploratory techniques for describing and modeling spatial distributions, patterns, processes, and relationships.


coincidence
connectivity
area
proximity
orientation
length
coincidence
direction



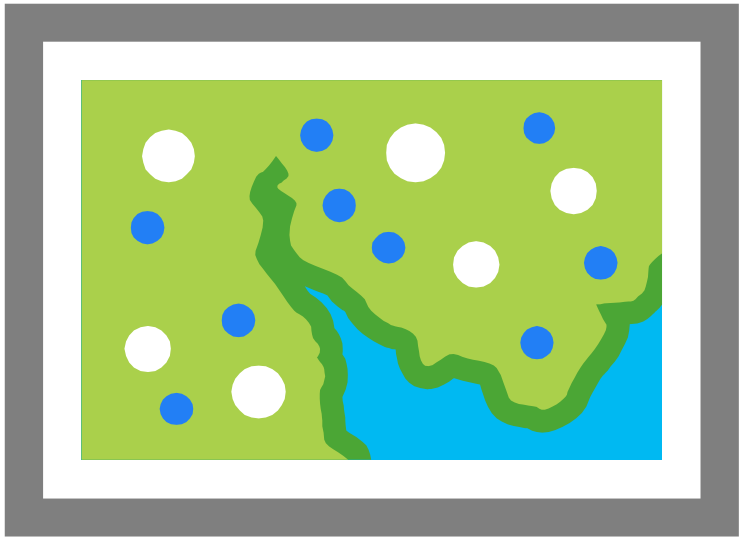
Spreadsheets

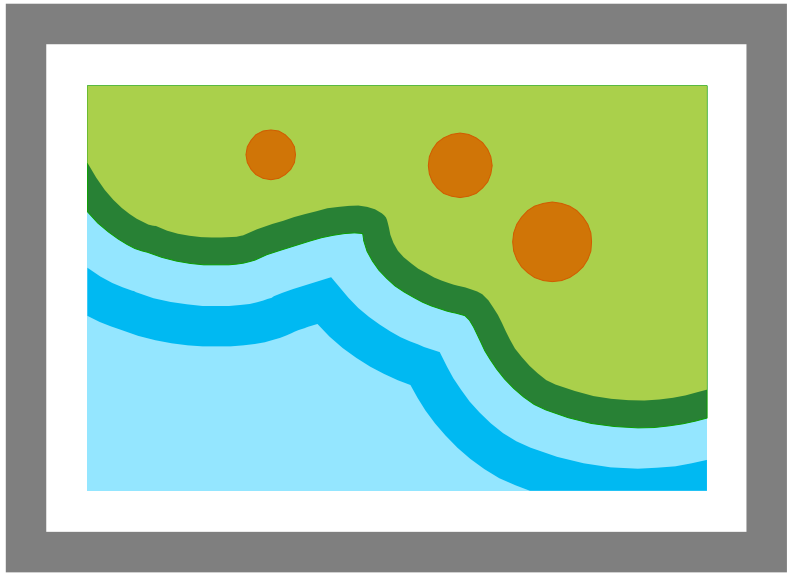
Data or Information?



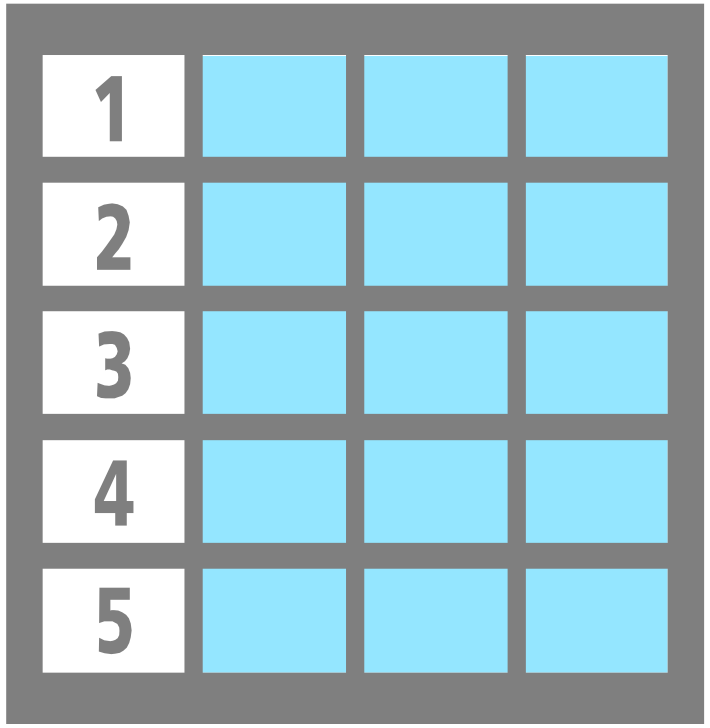
Maps

Data or Information?

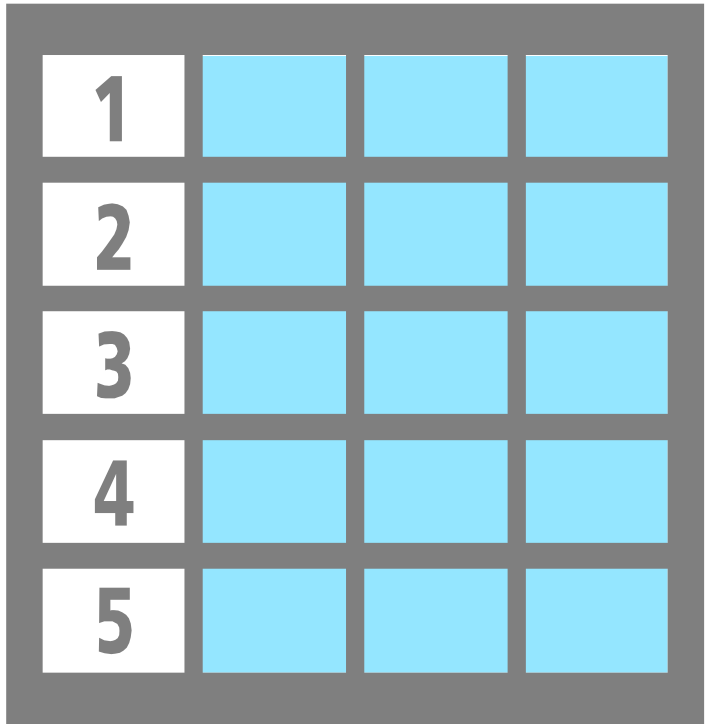


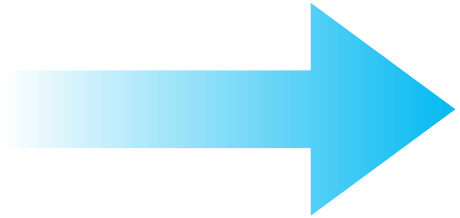


When you look at a spreadsheet...



You ask for more

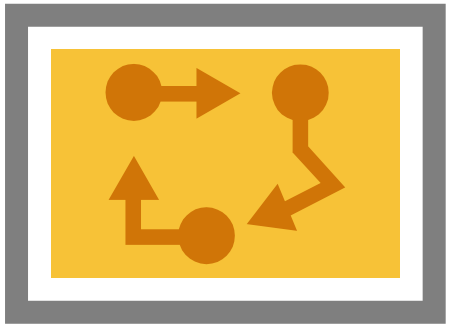


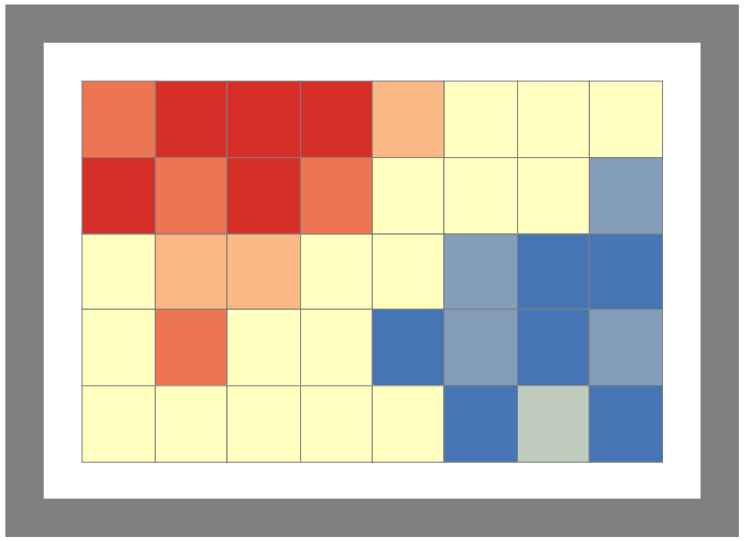


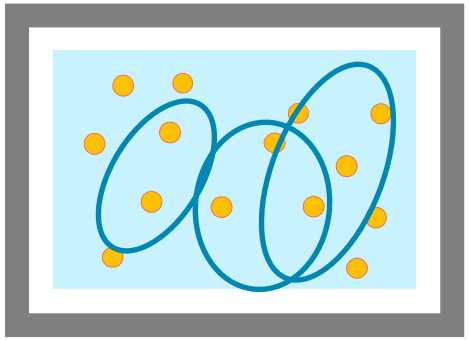
- Mean
- Standard Deviations
- Min and Max
- …

Same goes for maps!

We can do more







- Spatial Statistics Tools
  - Analyzing Patterns
    - Average Nearest Neighbor
    - High/Low Clustering (Getis-Ord General G)
    - Incremental Spatial Autocorrelation
    - Multi-Distance Spatial Cluster Analysis (Ripleys K Function)
    - Spatial Autocorrelation (Morans I)
  - Mapping Clusters
    - Cluster and Outlier Analysis (Anselin Local Morans I)
    - Grouping Analysis
    - Hot Spot Analysis (Getis-Ord Gi*)
    - Optimized Hot Spot Analysis
    - Similarity Search
  - Measuring Geographic Distributions
    - Central Feature
    - Directional Distribution (Standard Deviational Ellipse)
    - Linear Directional Mean
    - Mean Center
    - Median Center
    - Standard Distance
  - Modeling Spatial Relationships
    - Exploratory Regression
    - Generate Network Spatial Weights
    - Generate Spatial Weights Matrix
    - Geographically Weighted Regression
    - Ordinary Least Squares

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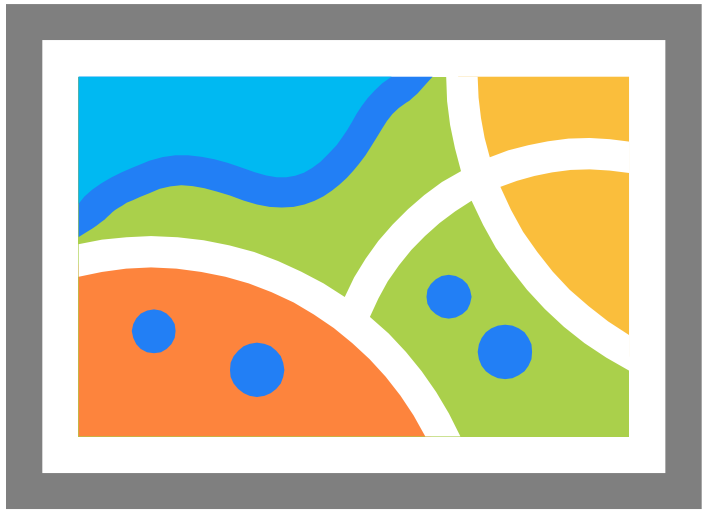
python script!

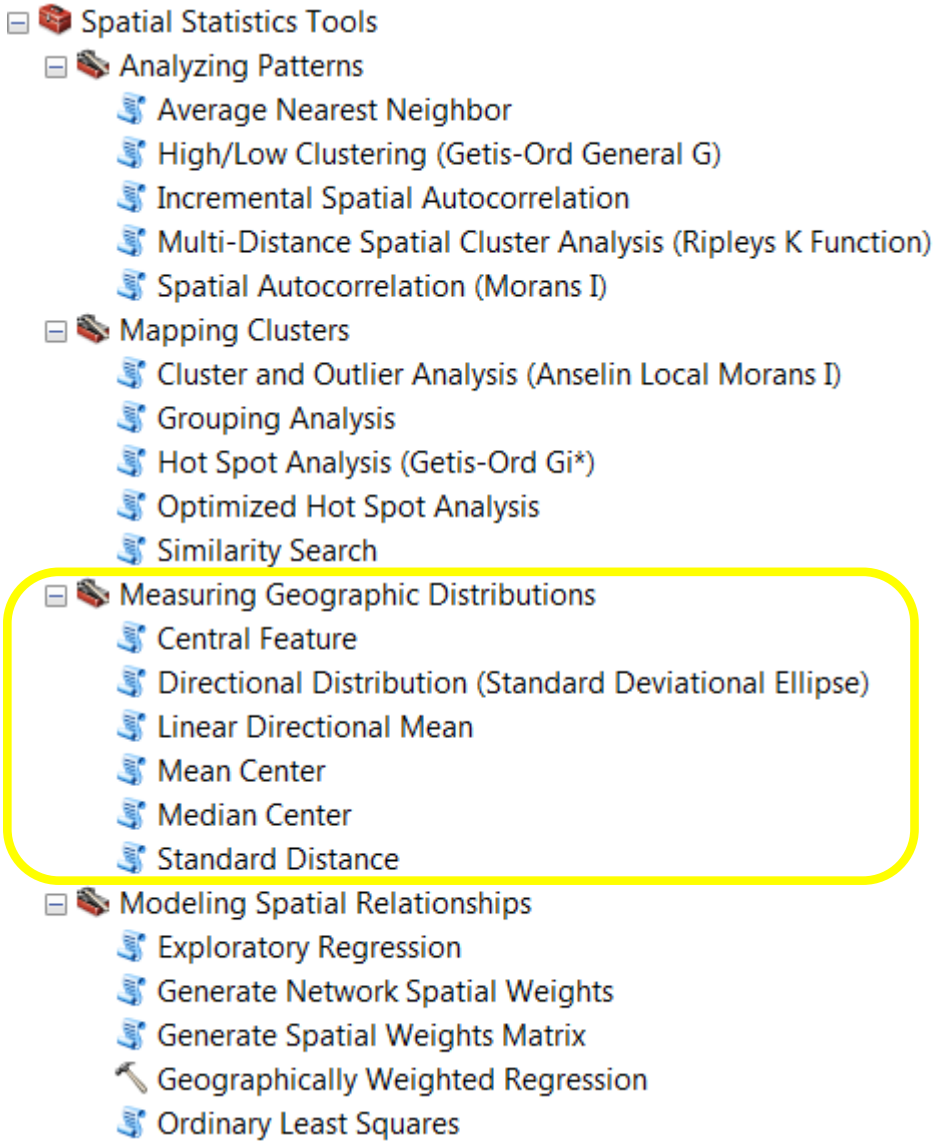
OptimizedHotSpotAnalysis - Notepad

File Edit Format View Help

```
        #### Remove Locations Outside Boundary FC ####
        featureLayer = "ClippedPointFC"
        DM.MakeFeatureLayer(tempFC, featureLayer)
        if self.boundaryFC:
            msg = ARCPY.GetIDMessage(84454)
            ARCPY.SetProgressor("default", msg)
            DM.SelectLayerByLocation(featureLayer, "INTERSECT",
                                     self.boundaryFC, "#",
                                     "NEW_SELECTION")
            DM.SelectLayerByLocation(featureLayer, "INTERSECT",
                                    "#", "#", "SWITCH_SELECTION")
            DM.DeleteFeatures(featureLayer)
        else:
            if additionalZeroDistScale == "ALL":
                msg = ARCPY.GetIDMessage(84455)
                ARCPY.SetProgressor("default", msg)
                DM.SelectLayerByAttribute(featureLayer, "NEW_SELECTION",
                                          '"Join_Count" = 0')
                DM.DeleteFeatures(featureLayer)

            else:
                distance = additionalZeroDistScale * fish.quadLength
                distanceStr = self.ssdo.distanceInfo.linearUnitString(distance,
                                                                      convert = True)
                nativeStr = self.ssdo.distanceInfo.printDistance(distance)
                msg = "Removing cells further than %s from input pointsd...."
                ARCPY.AddMessage(msg % nativeStr)
                DM.SelectLayerByLocation(featureLayer, "INTERSECT",
                                         self.ssdo.inputFC, distanceStr,
                                         "NEW_SELECTION")
                DM.SelectLayerByLocation(featureLayer, "INTERSECT",
                                         "#", "#", "SWITCH_SELECTION")
                DM.DeleteFeatures(featureLayer)

        DM.Delete(featureLayer)
        del collSSDO

        ARCPY.env.extent = oldExtent
```

Measuring Geographic Distributions

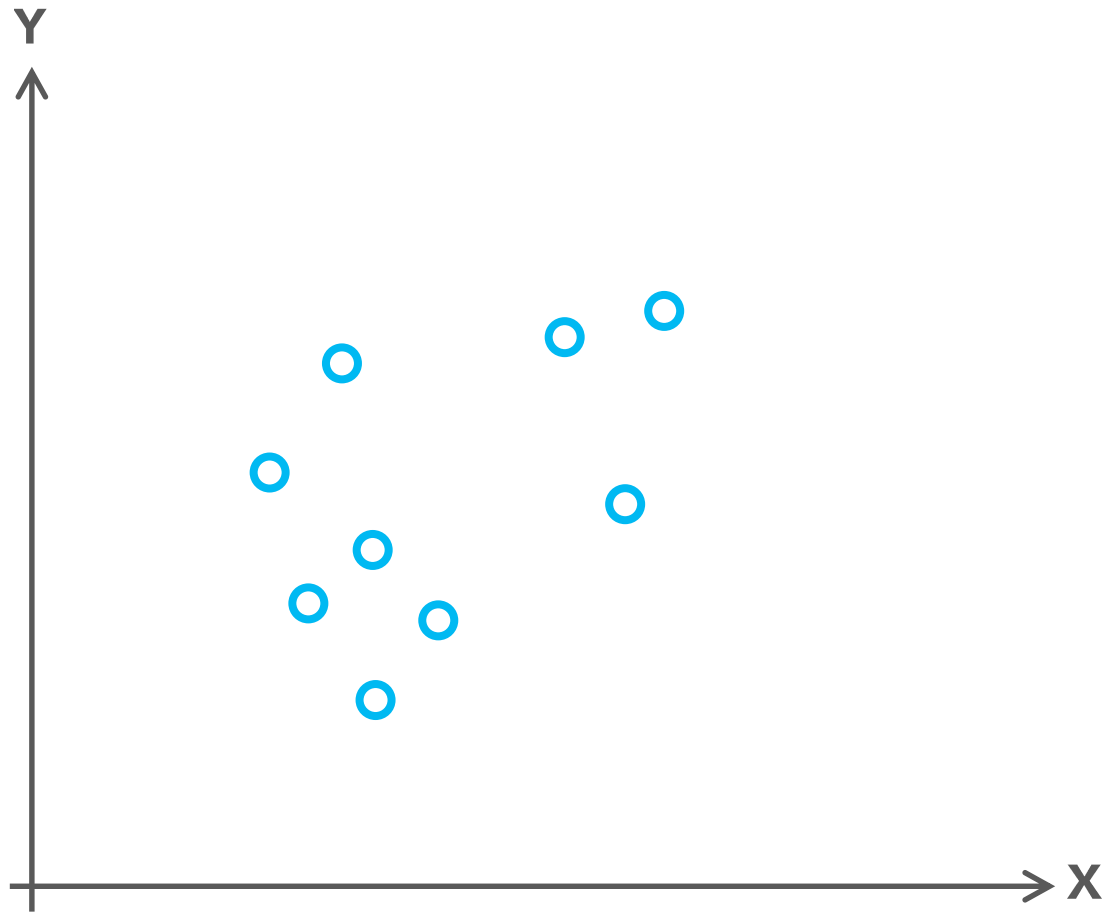


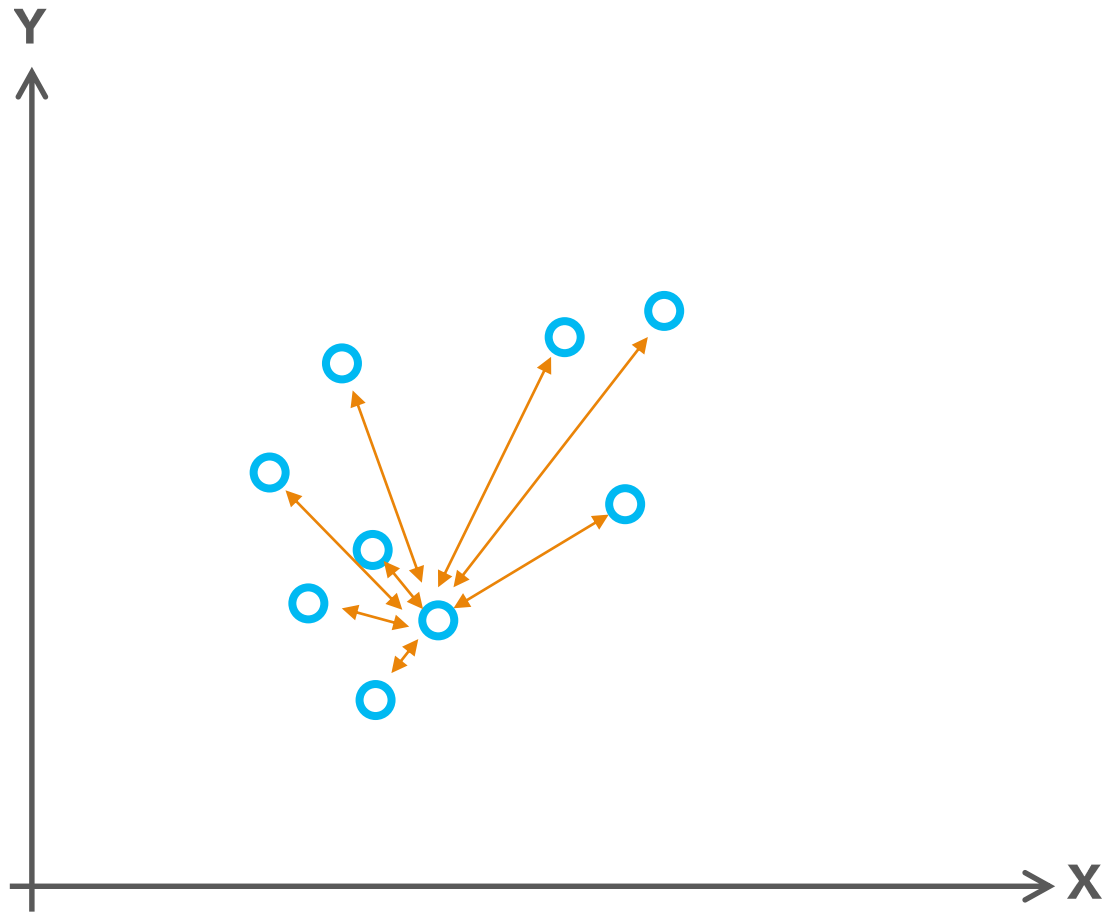


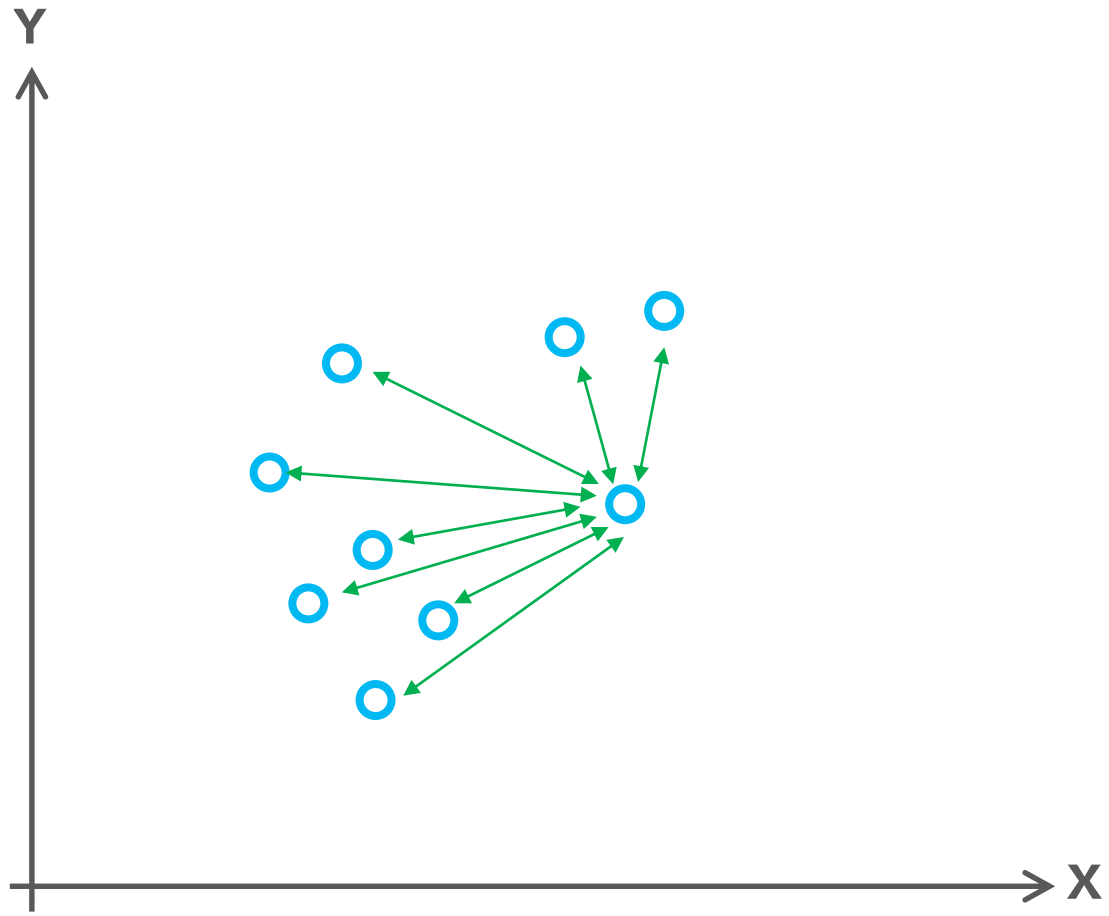
These tools address questions such as Where's the center? What's the shape and orientation? How dispersed are the features?

Central Feature

identifies the most centrally located feature in a point, line, or polygon feature class


Y
X


Y
X


Y
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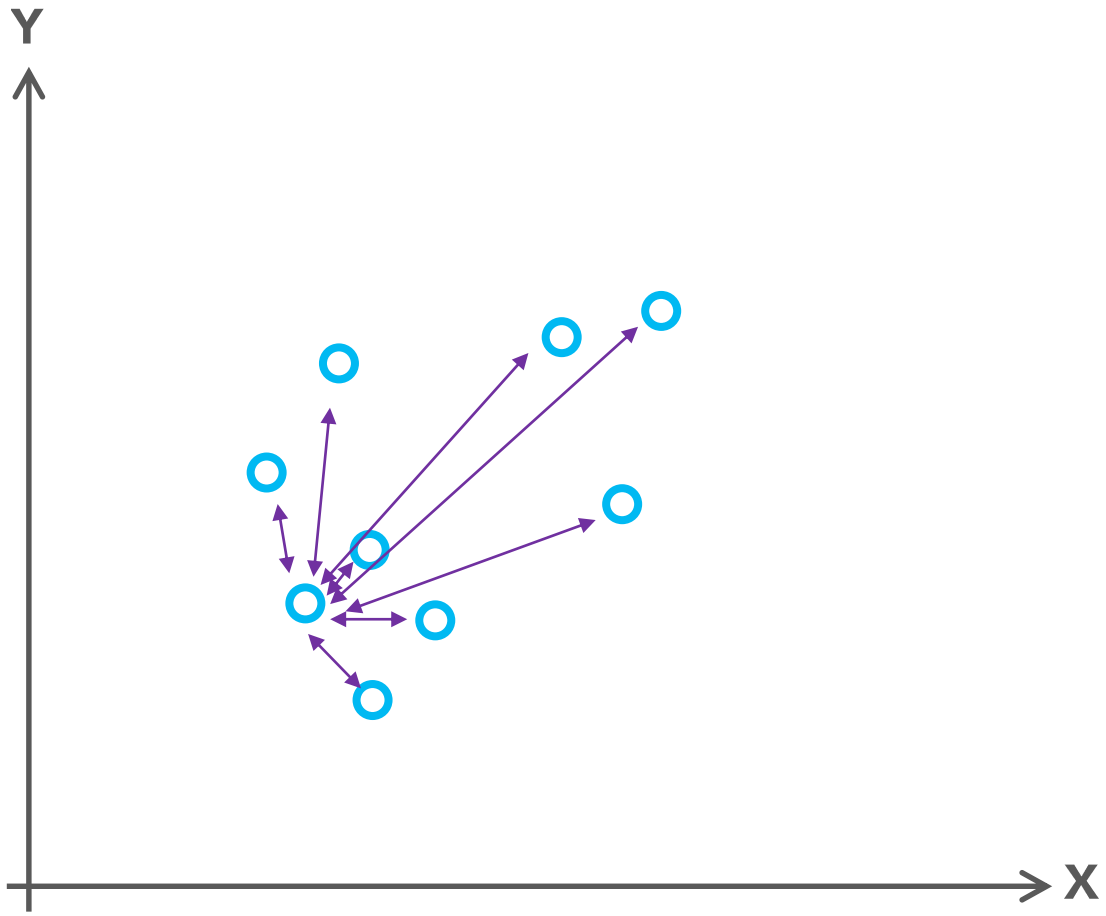
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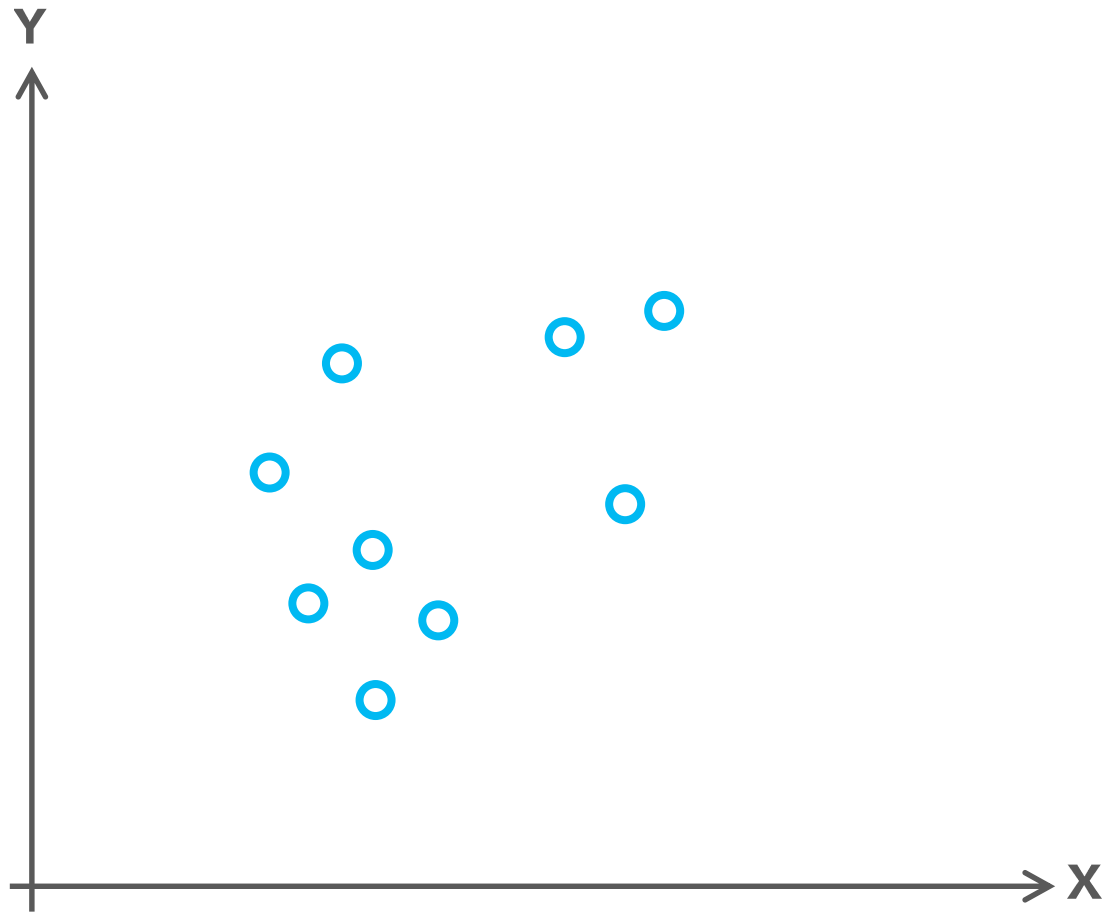
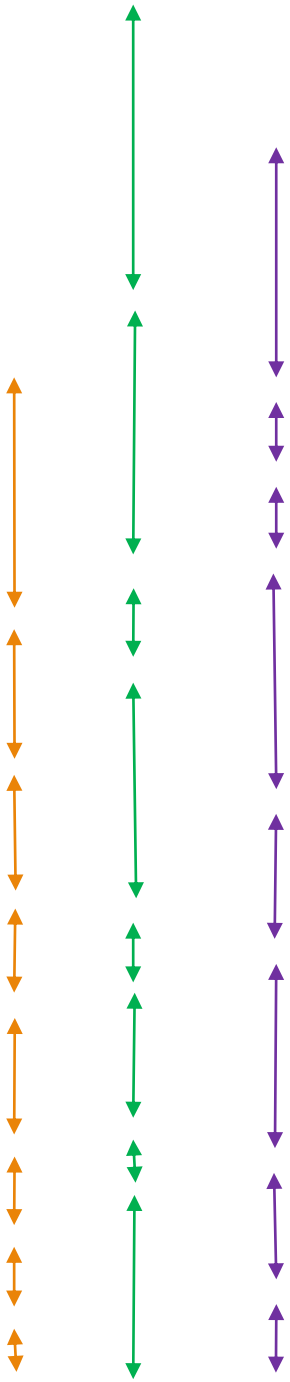
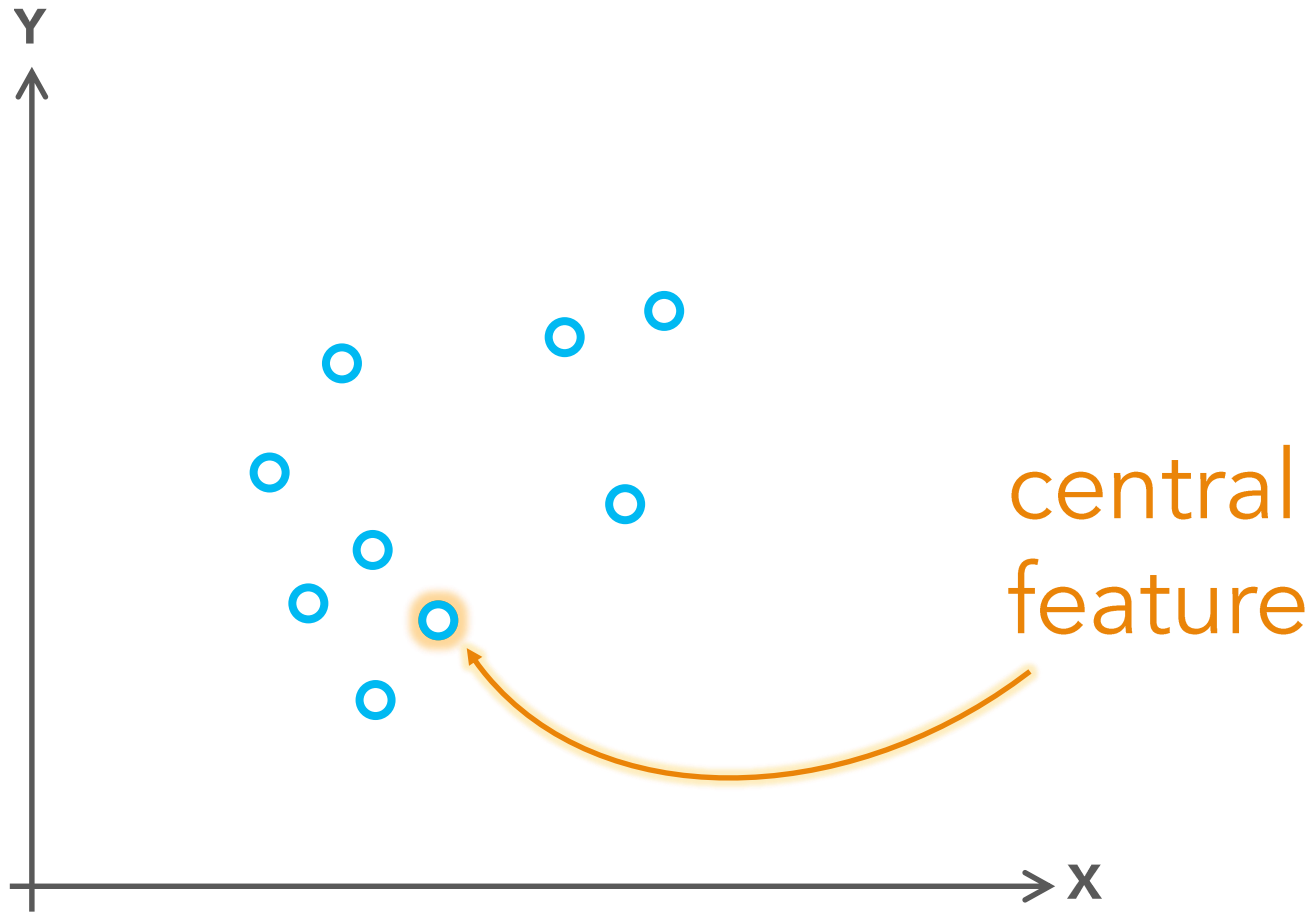
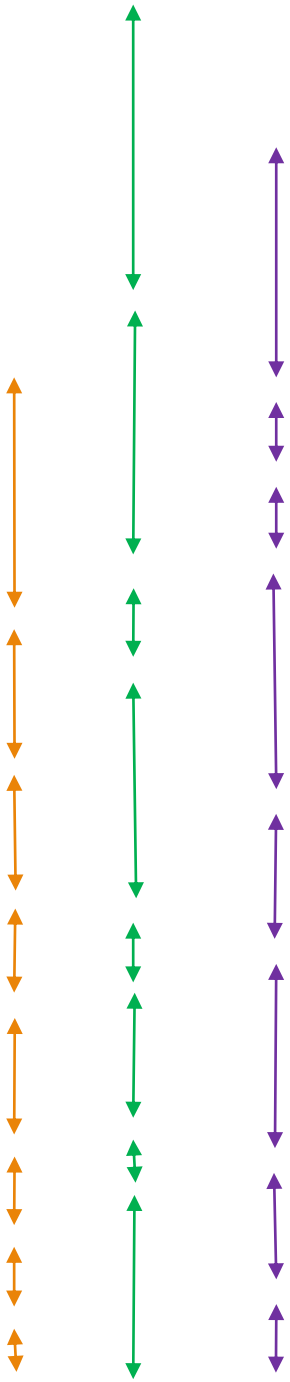

Y
X

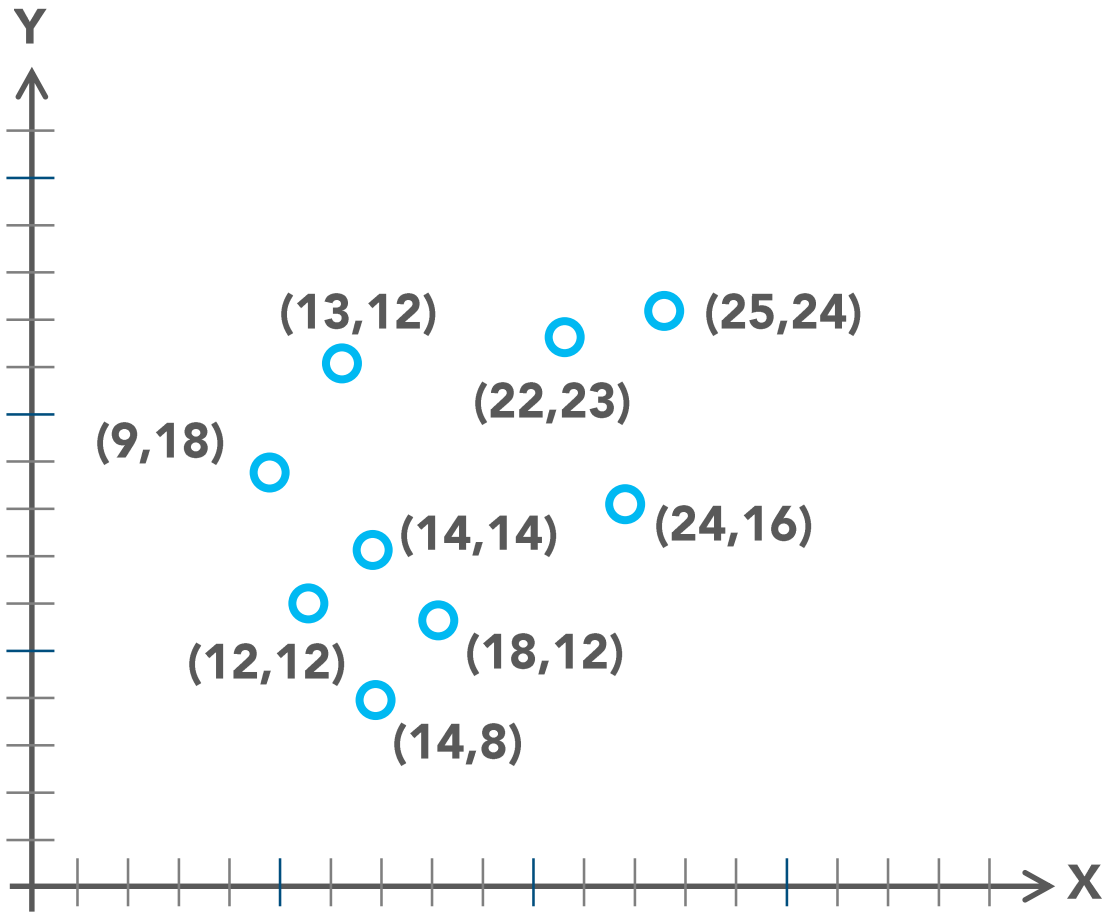


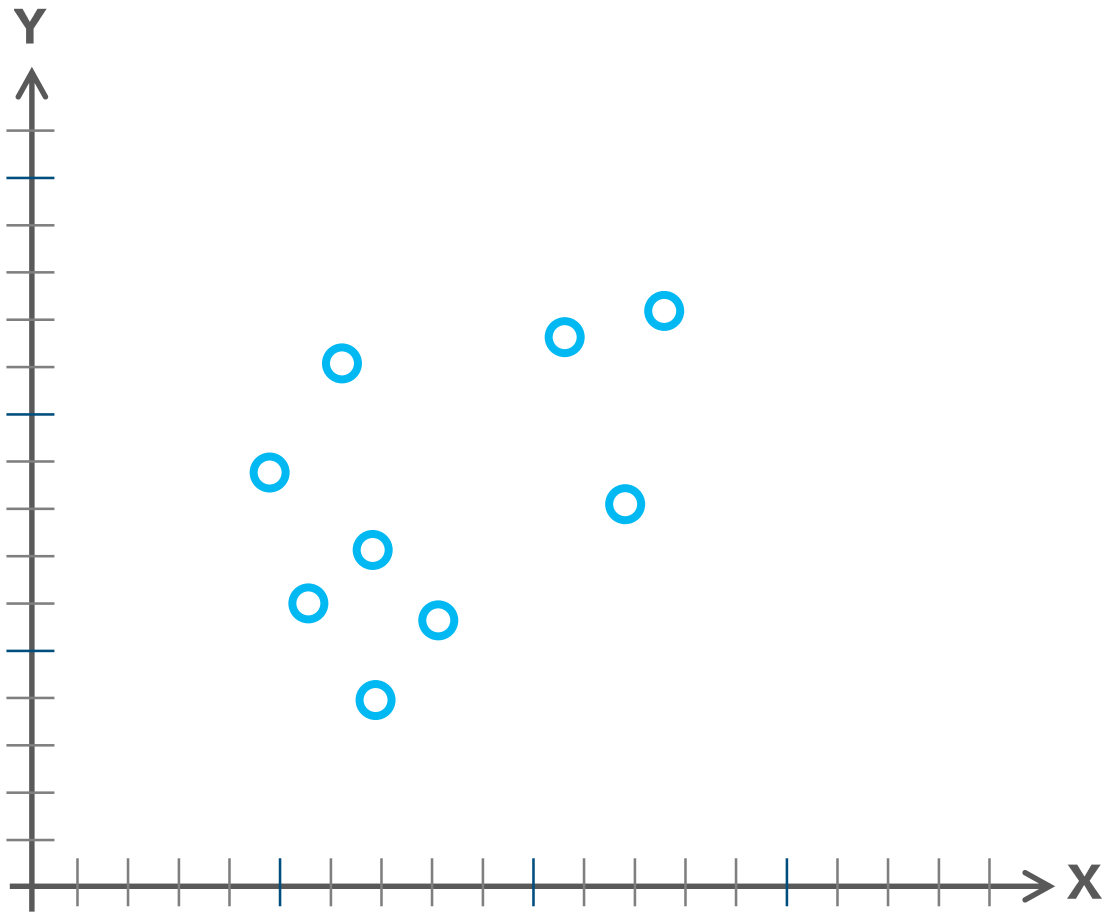

Y
X
central
feature



Mean Center

identifies the geographic center (or the center of concentration) for a set of features


Y
X
(13,12)
(25,24)
(22,23)
(9,18)
(14,14)
(24,16)
(12,12)
(18,12)
(14,8)



(14,14)

(13,12)

(25,24)

(24,16)

(22,23)

(18,12)

(12,12)

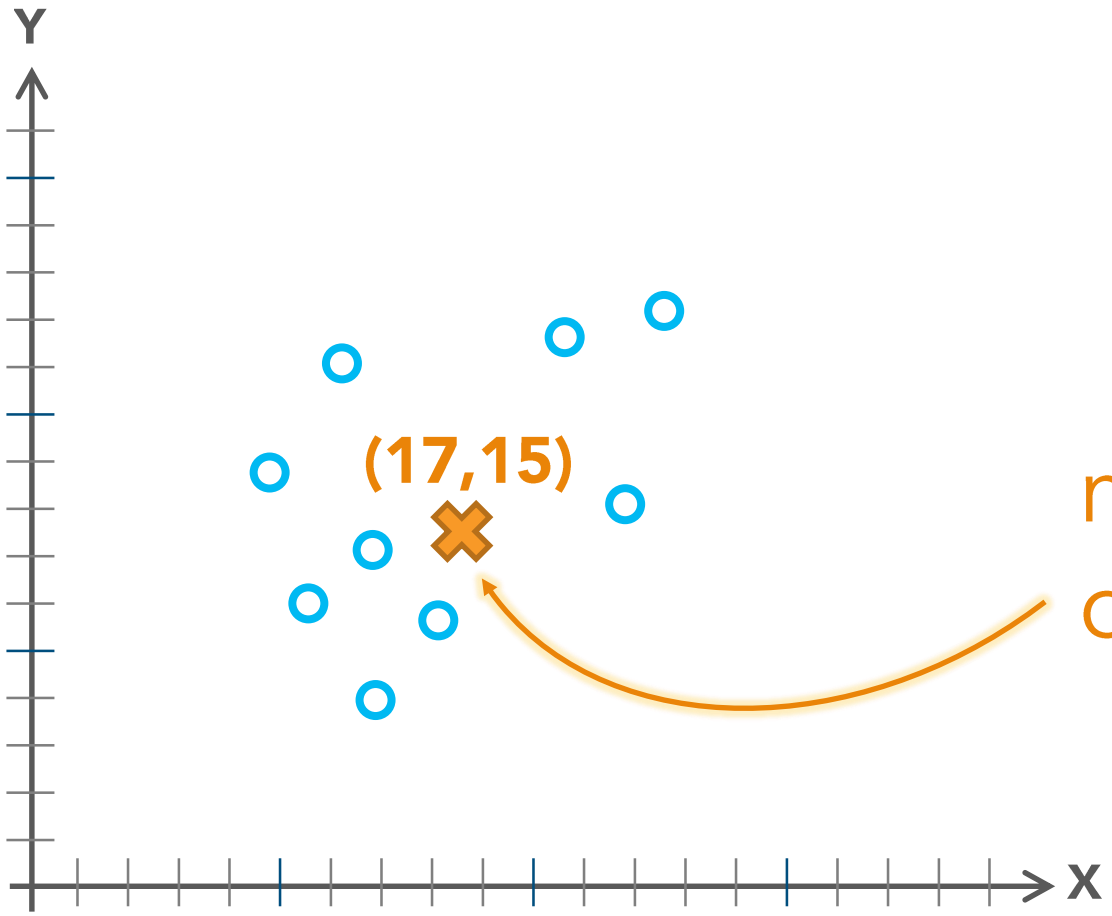
(14,8)

(9,18)

---

mean =

(17,15)



(14,14)

(13,12)

(25,24)

(24,16)

(22,23)

(18,12)

(12,12)

(14,8)

(9,18)

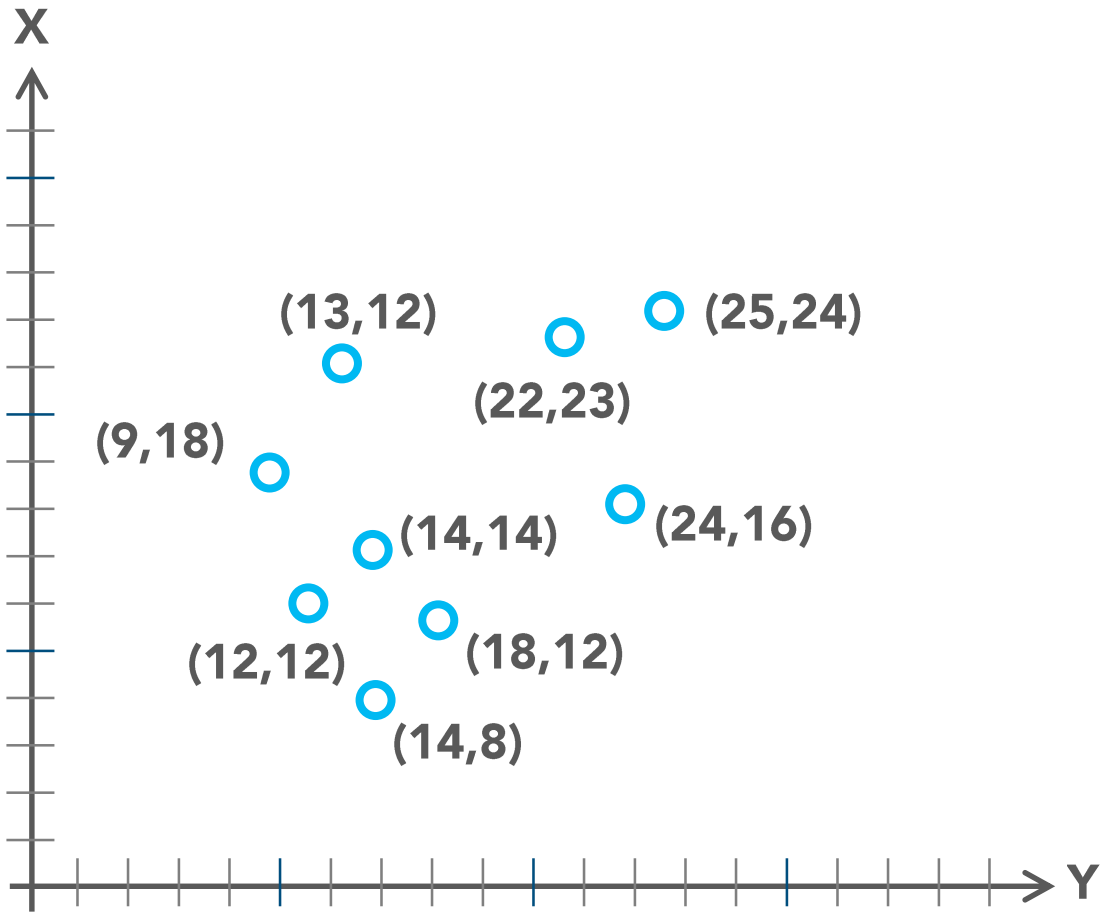
---

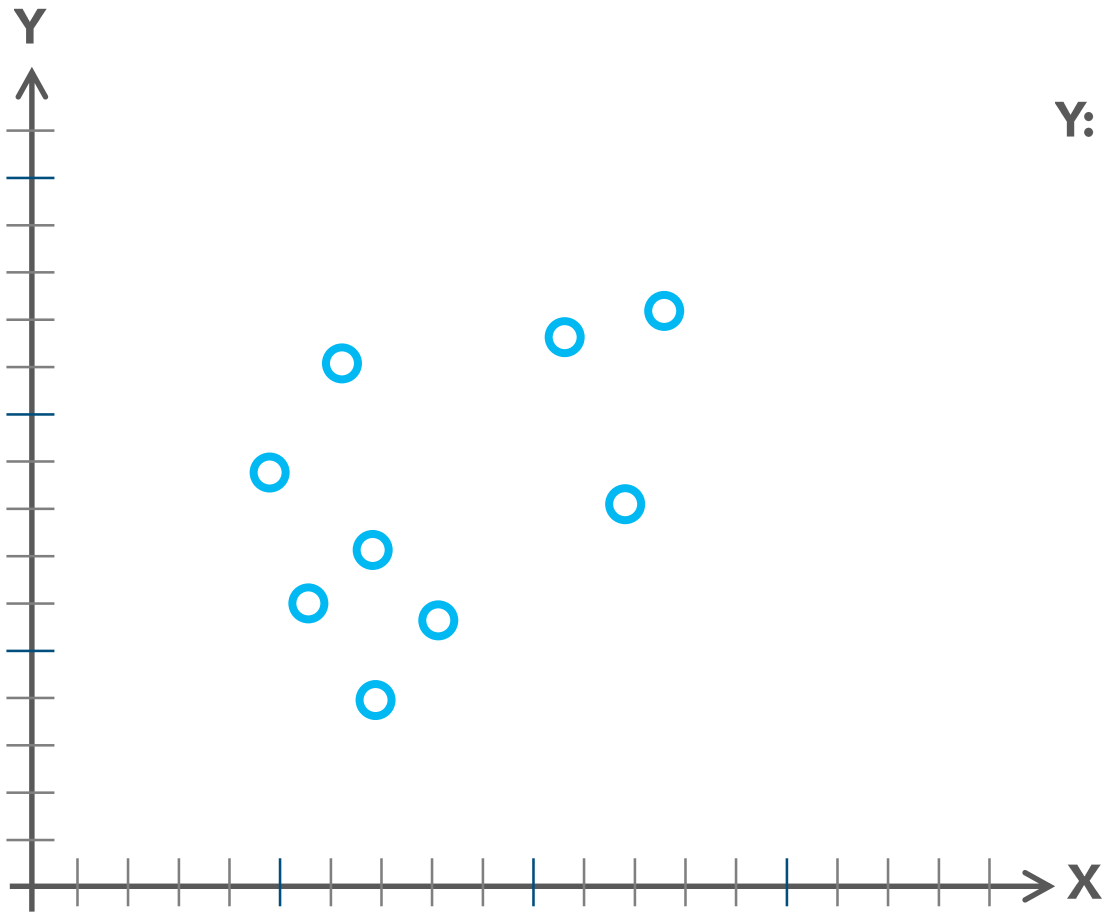
mean =

(17,15)

Median Center

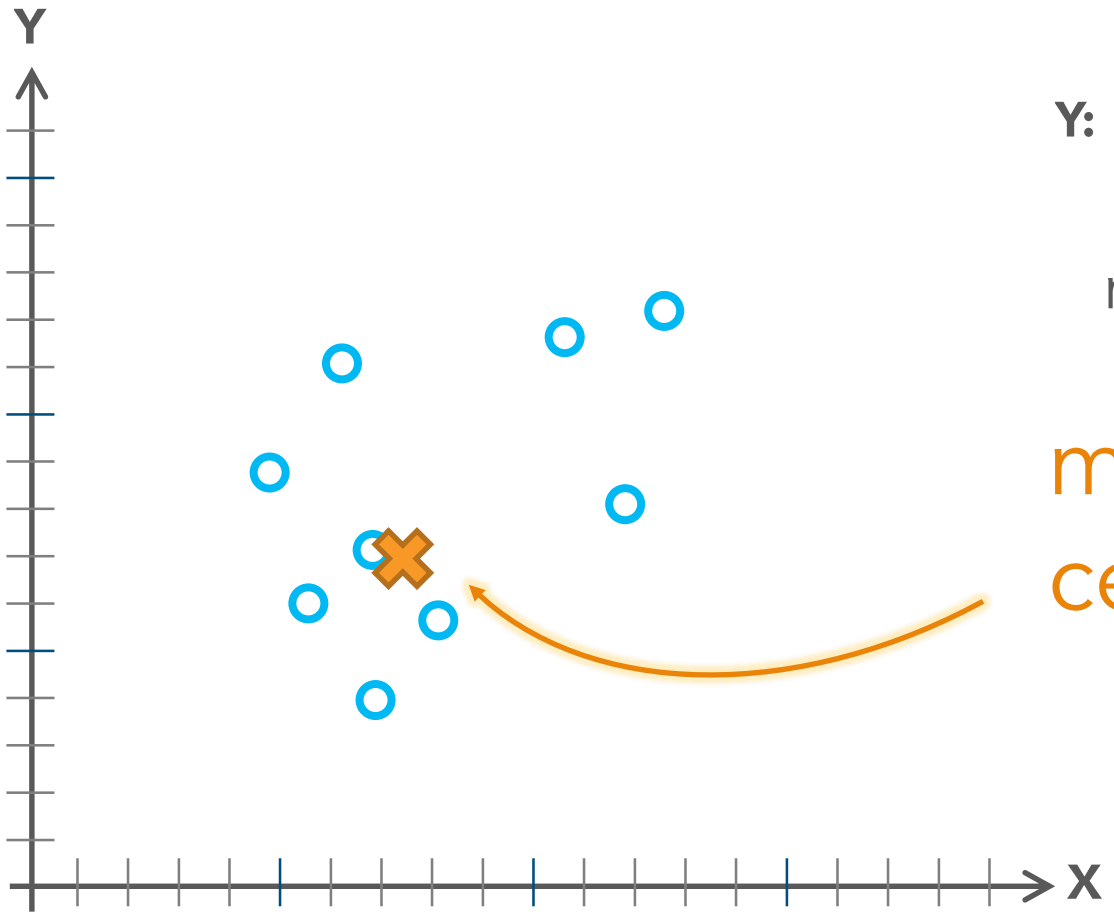
identifies the location that minimizes overall Euclidean distance to the features in a dataset


X
Y
(13,12)
(25,24)
(22,23)
(9,18)
(14,14)
(24,16)
(12,12)
(18,12)
(14,8)



X: 25 . 24 . 22 . 18 . 14 . 14 . 13 . 12 . 9

Y: 24 . 23 . 18 . 16 . 14 . 12 . 12 . 12 . 8



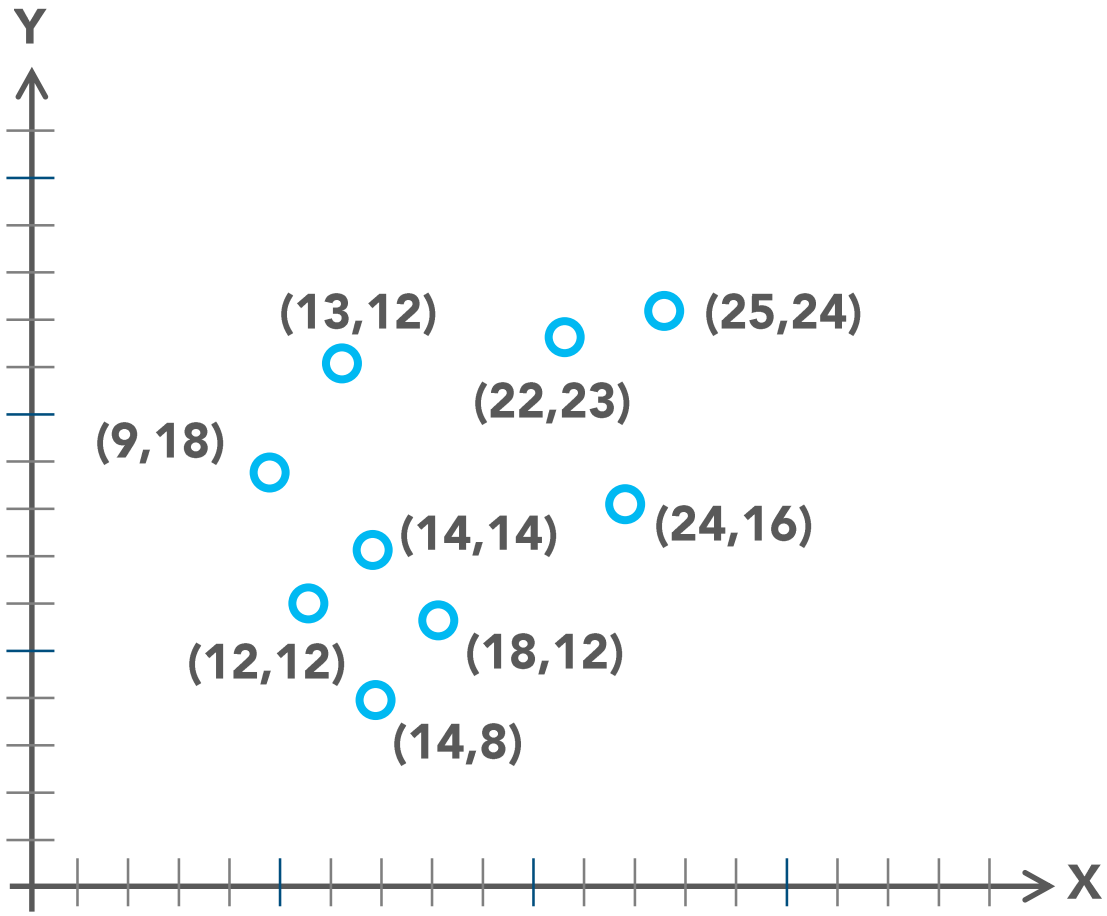
X: ~~25~~ . ~~24~~ . ~~22~~ . ~~18~~ . 14 . ~~14~~ . ~~13~~ . ~~12~~ . ~~8~~

Y: ~~24~~ . ~~23~~ . ~~18~~ . ~~16~~ . 14 . ~~12~~ . ~~12~~ . ~~12~~ . ~~8~~

median = **(14,14)**

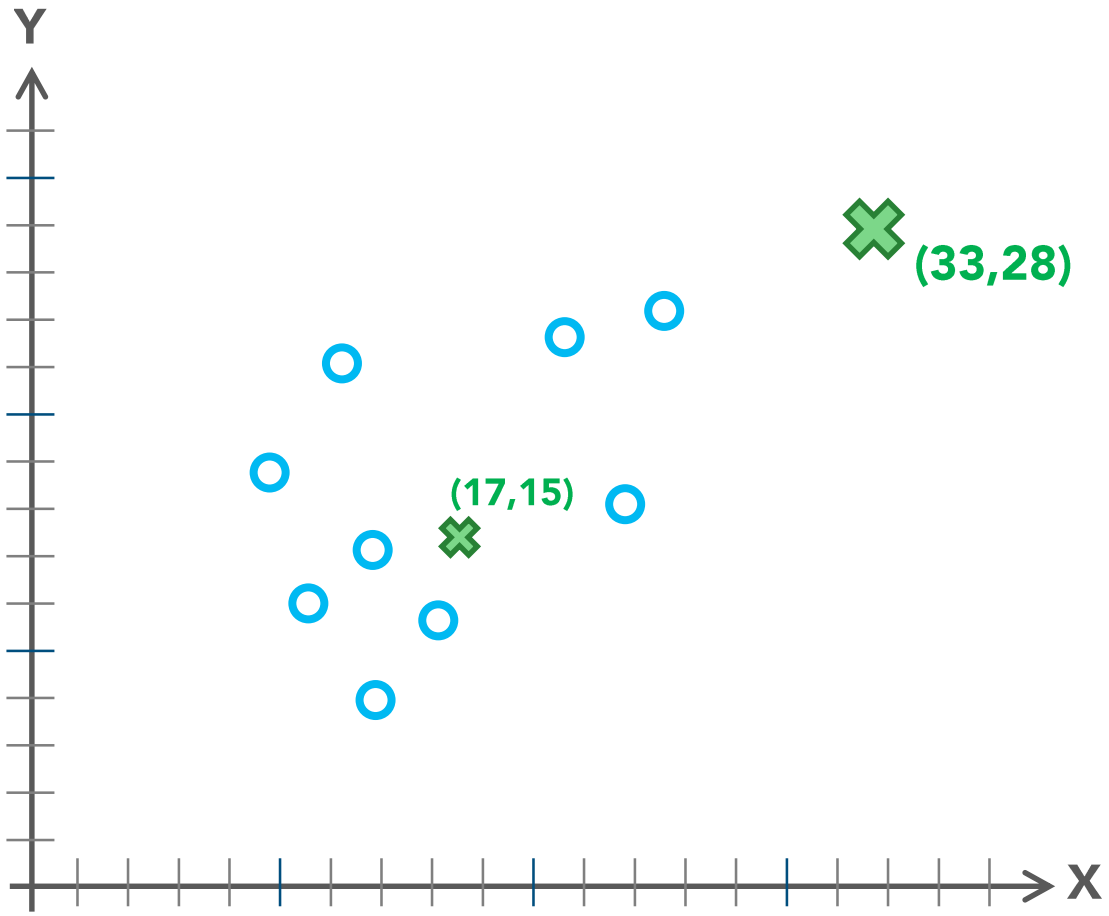
median
center

Mean vs Median?



outlier

(176,138)



(14,14)

(13,12)

(25,24)

(24,16)

(22,23)

(18,12)

(12,12)

(14,8)

(9,18)

(74,38)

---

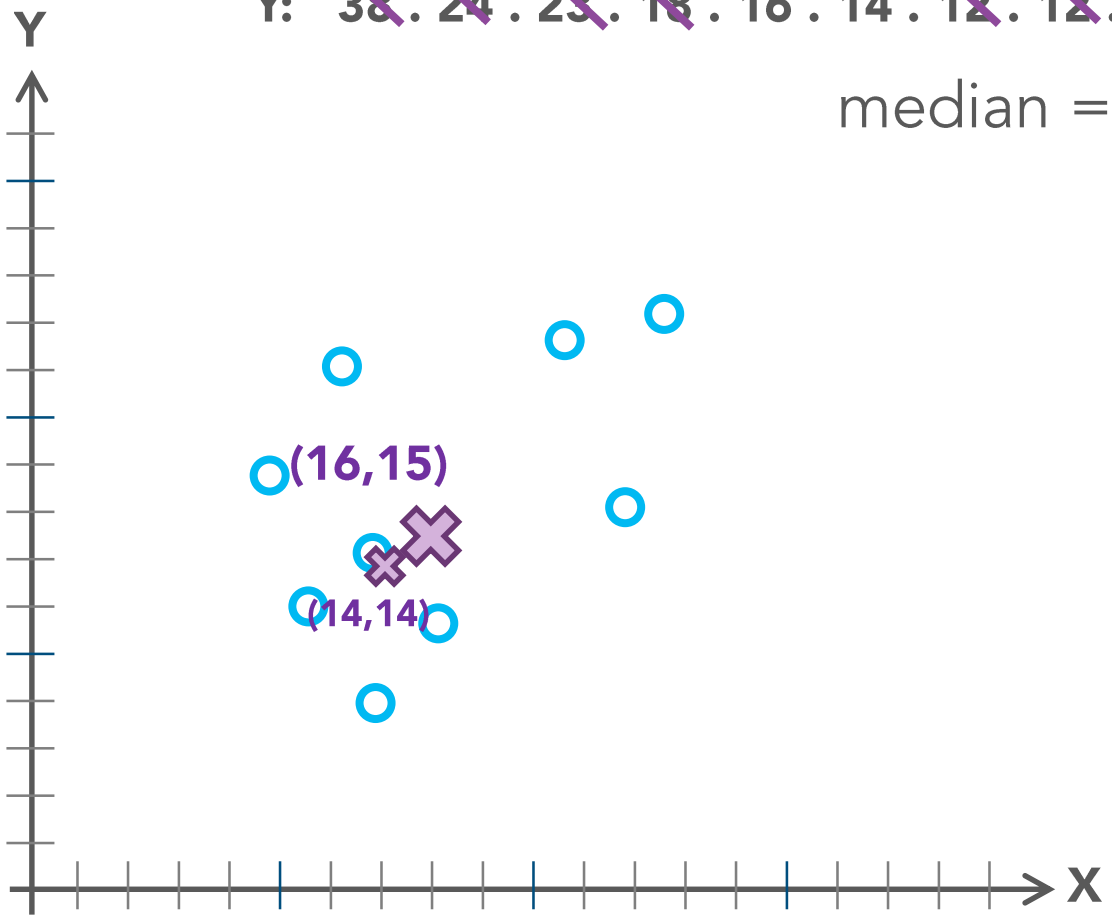
mean =

(33,28)

X: ~~74~~ . ~~25~~ . ~~24~~ . ~~22~~ . 18 . 14 . ~~14~~ . ~~13~~ . ~~12~~ . ~~9~~

Y: ~~38~~ . ~~24~~ . ~~23~~ . ~~18~~ . 16 . 14 . ~~12~~ . ~~12~~ . ~~12~~ . ~~8~~

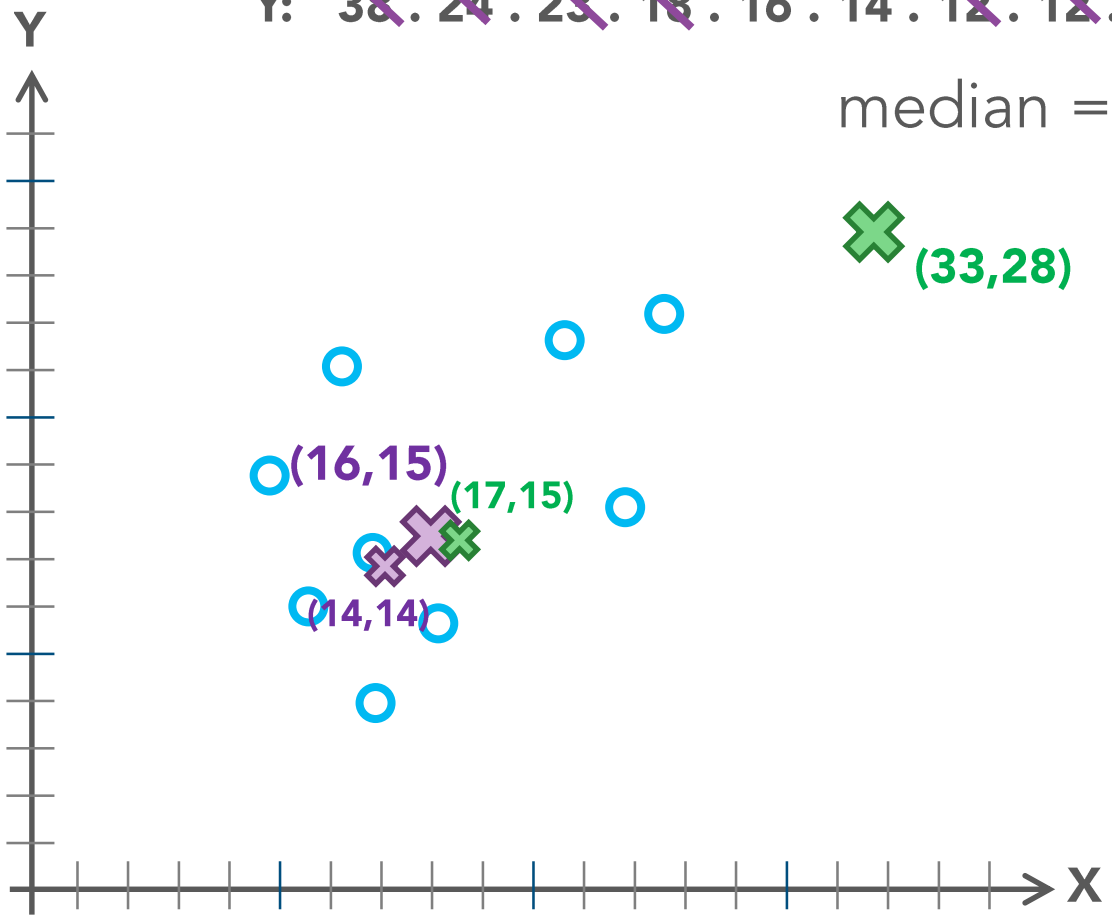
median = **(16,15)**



X: ~~74~~ . ~~25~~ . ~~24~~ . ~~22~~ . 18 . 14 . ~~14~~ . ~~13~~ . ~~12~~ . ~~9~~

Y: ~~38~~ . ~~24~~ . ~~23~~ . ~~18~~ . 16 . 14 . ~~12~~ . ~~12~~ . ~~12~~ . ~~8~~

median = **(16,15)**



(14,14)

(13,12)

(25,24)

(24,16)

(22,23)

(18,12)

(12,12)

(14,8)

(9,18)

(74,38)

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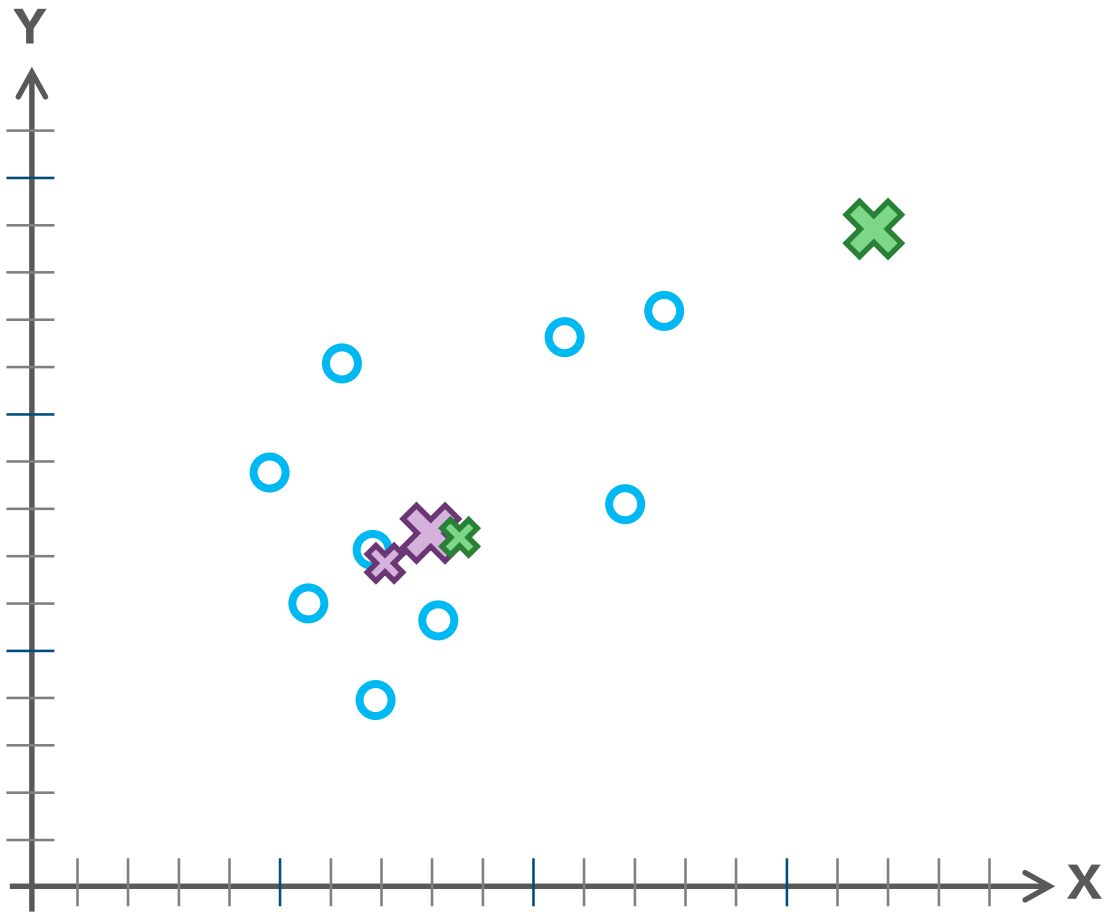
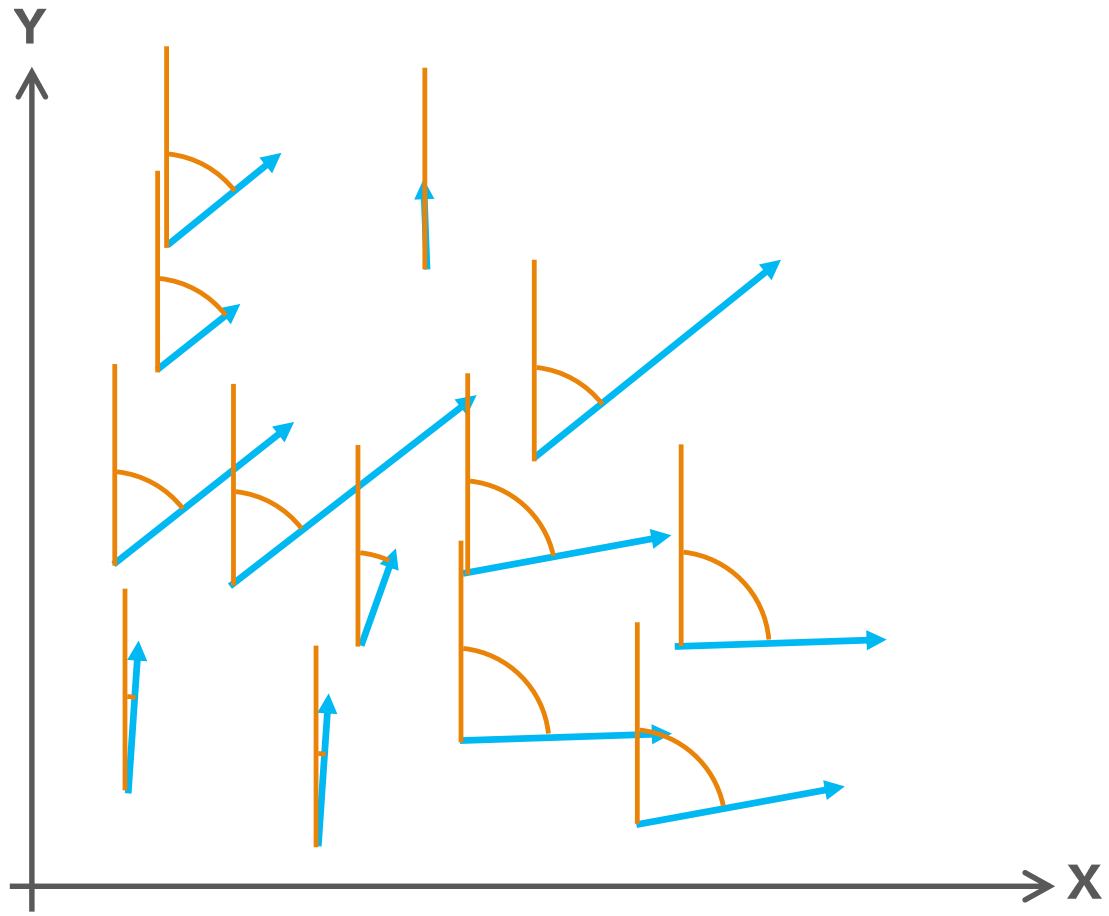
mean =

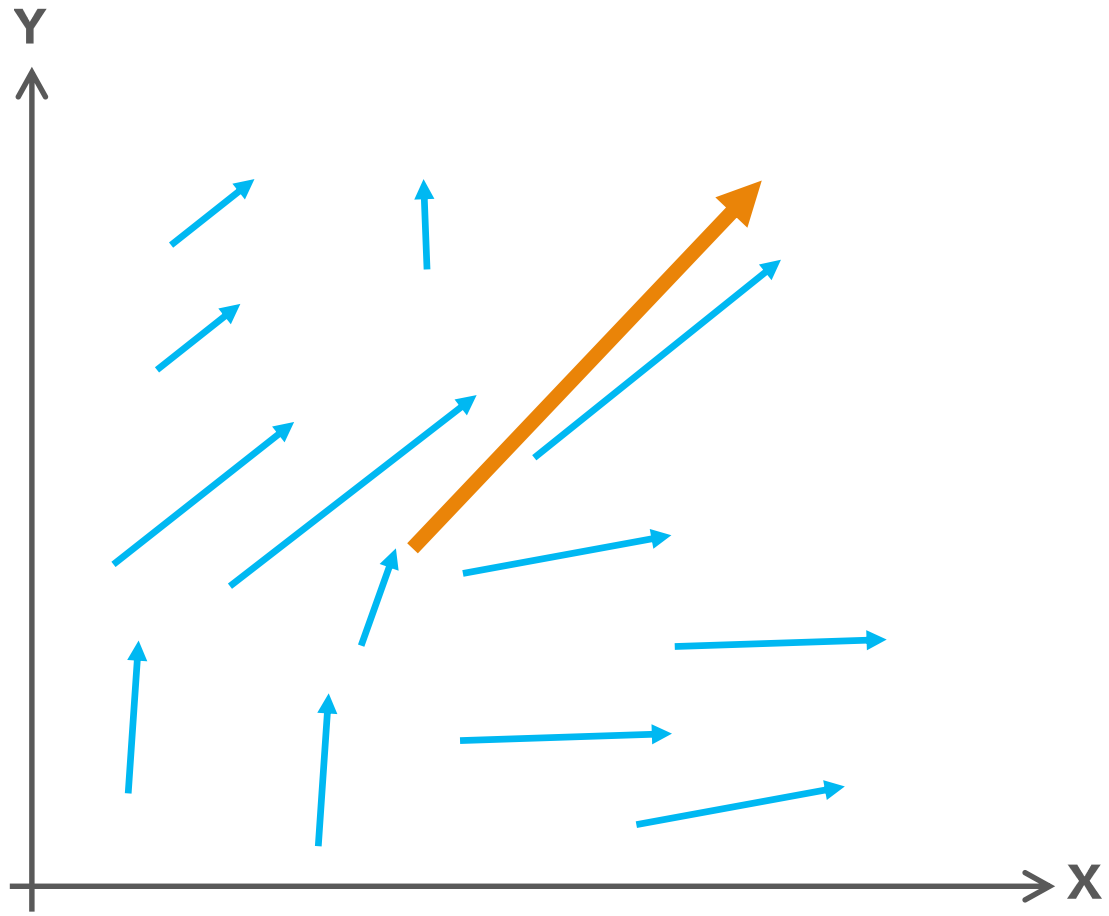
(33,28)


Y
X

Linear Directional Mean

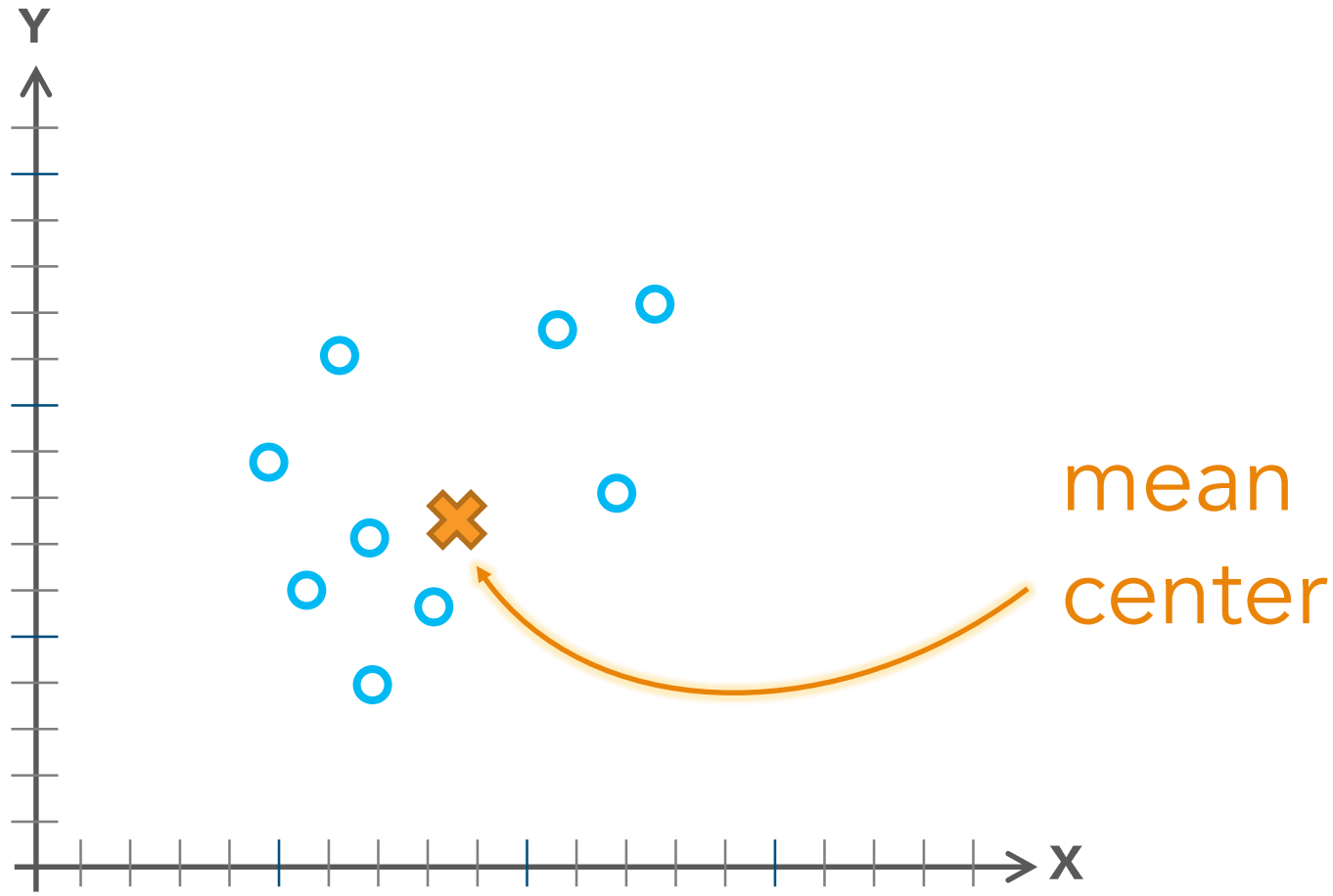
identifies the mean direction, length, and geographic center for a set of lines

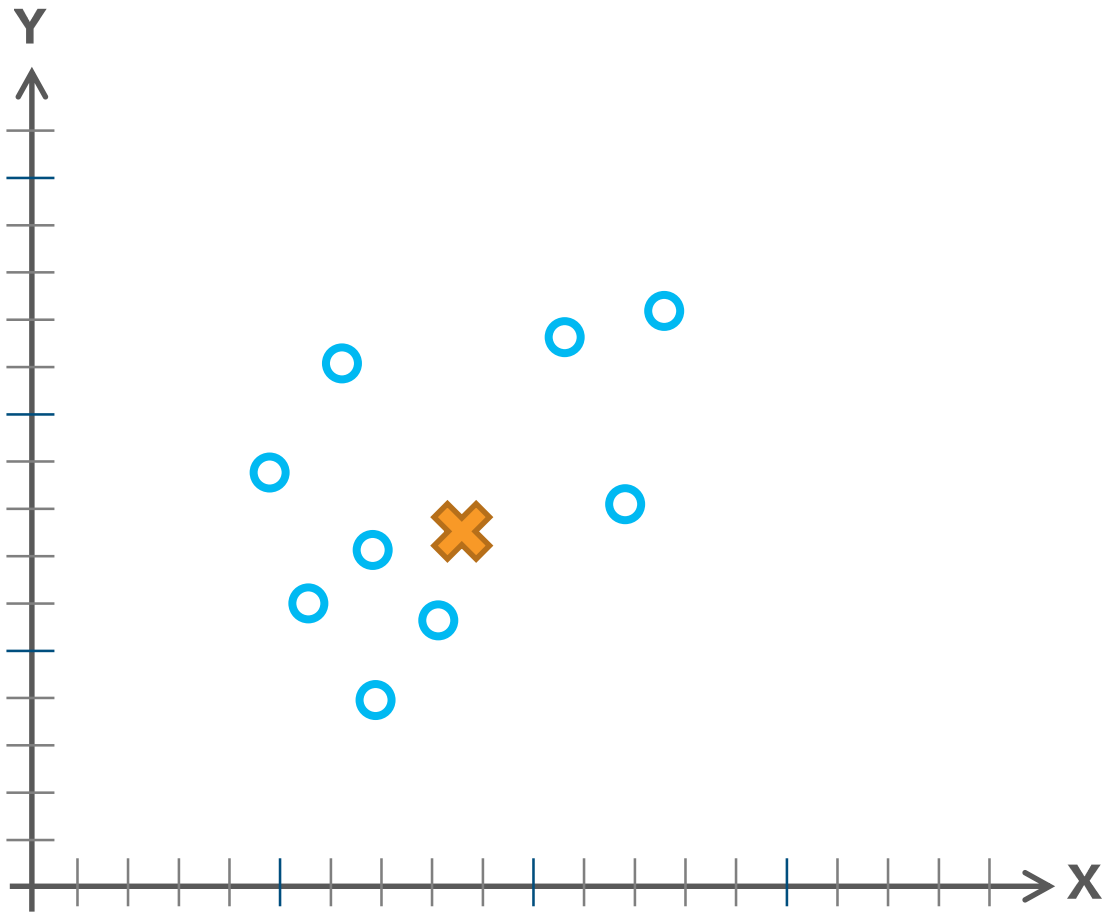

Y
X

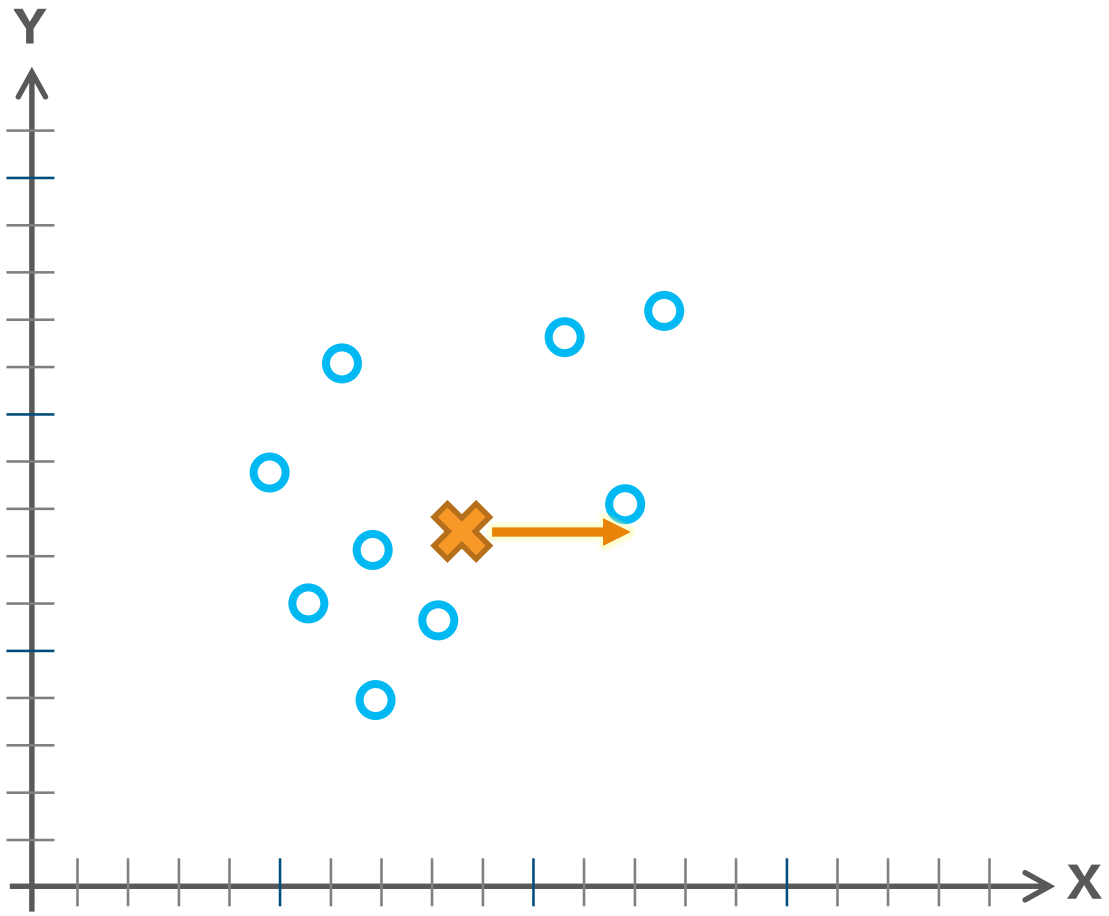

Y
X

Standard Distance

measures the degree to which features are concentrated or dispersed around the geometric mean center


Y
X
mean
center


Y
X


Y
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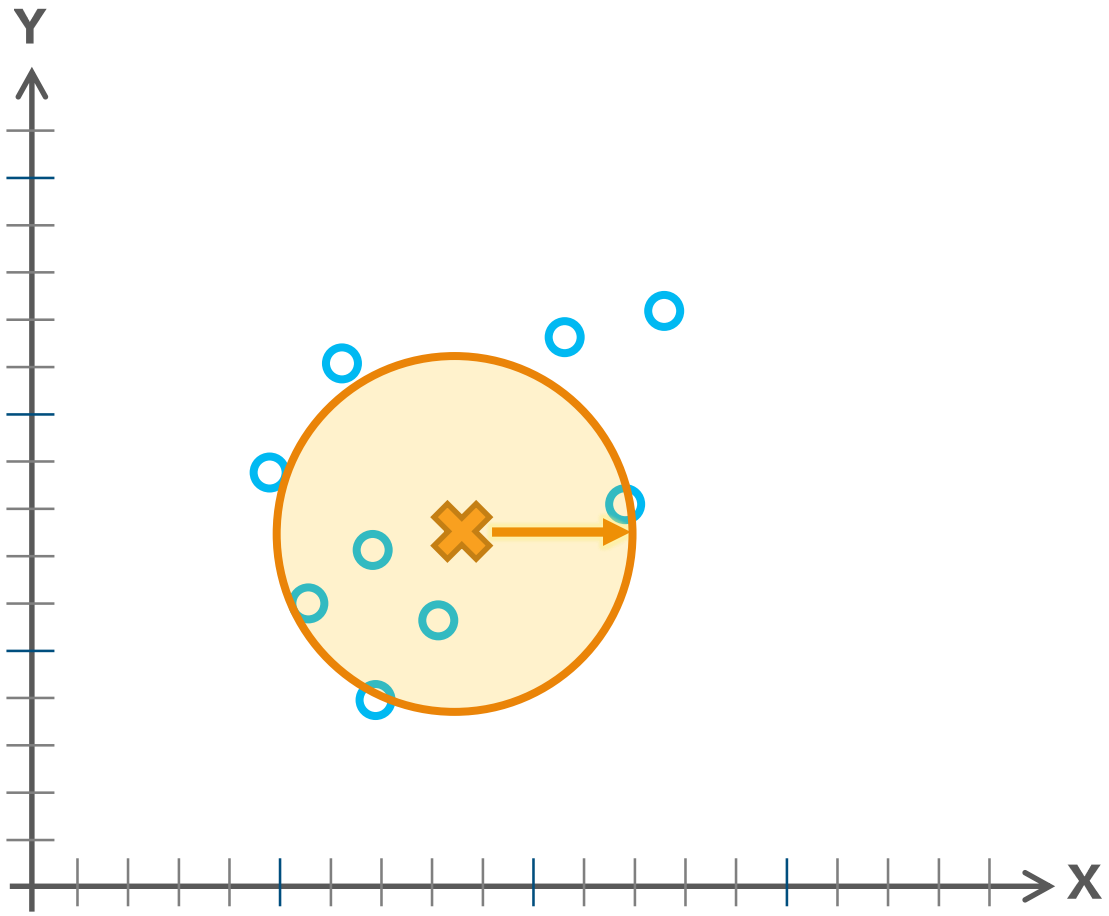
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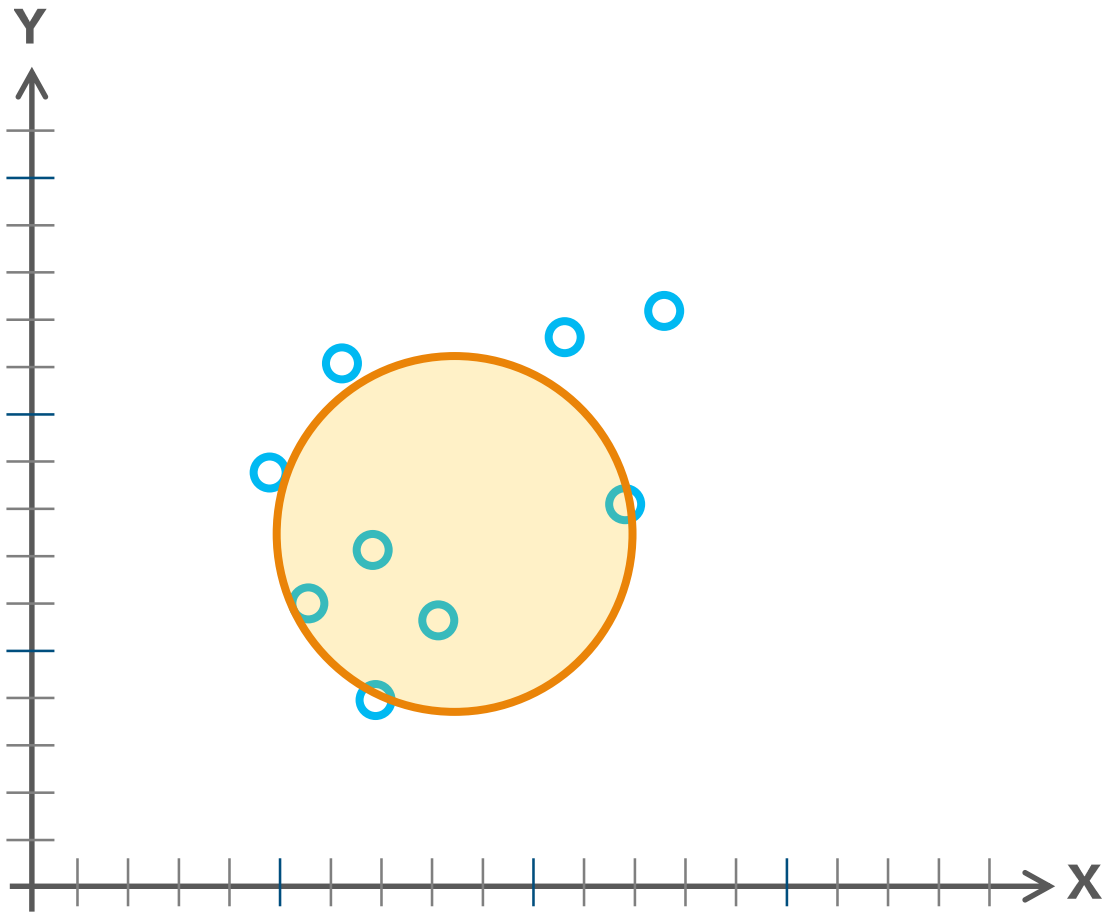
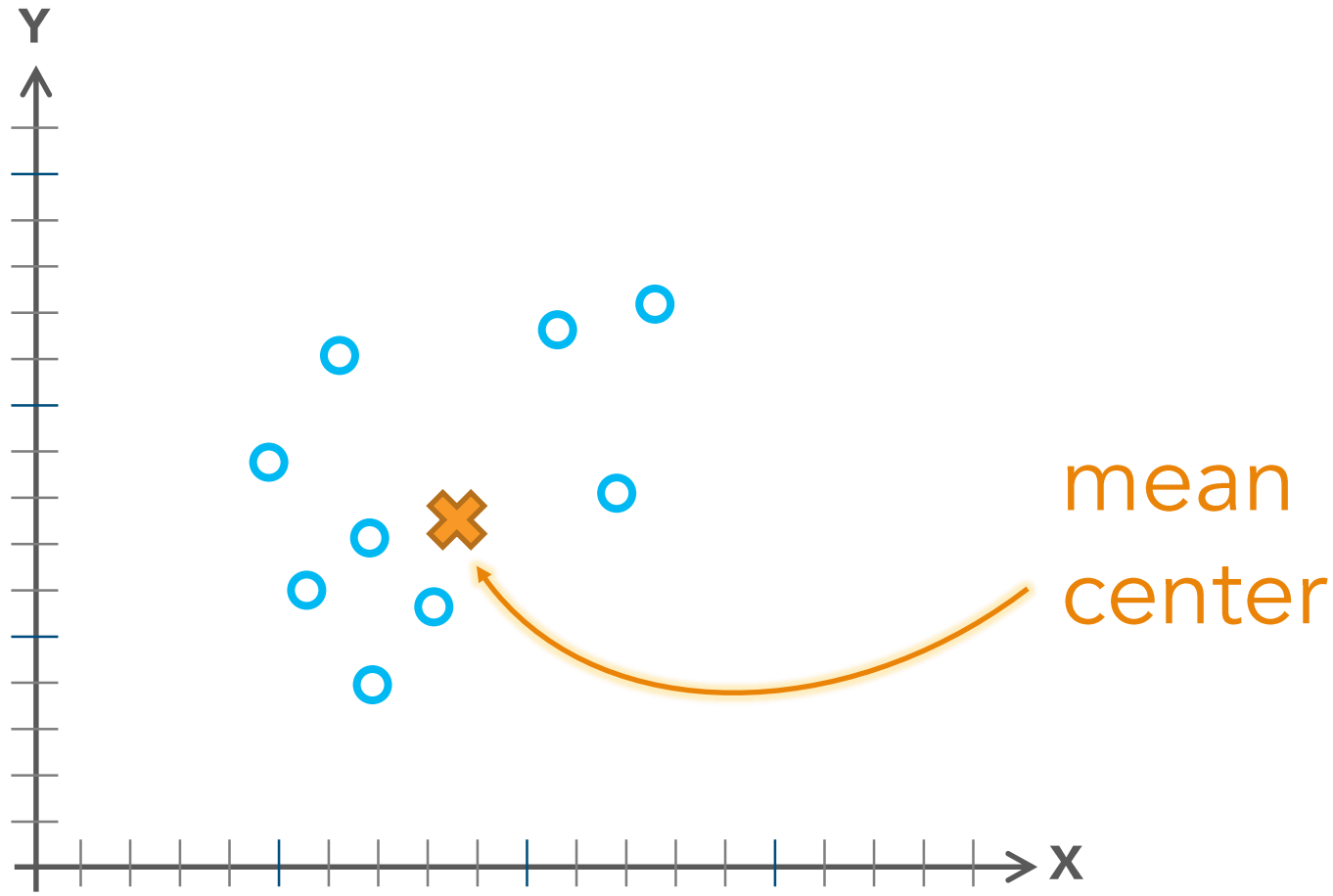
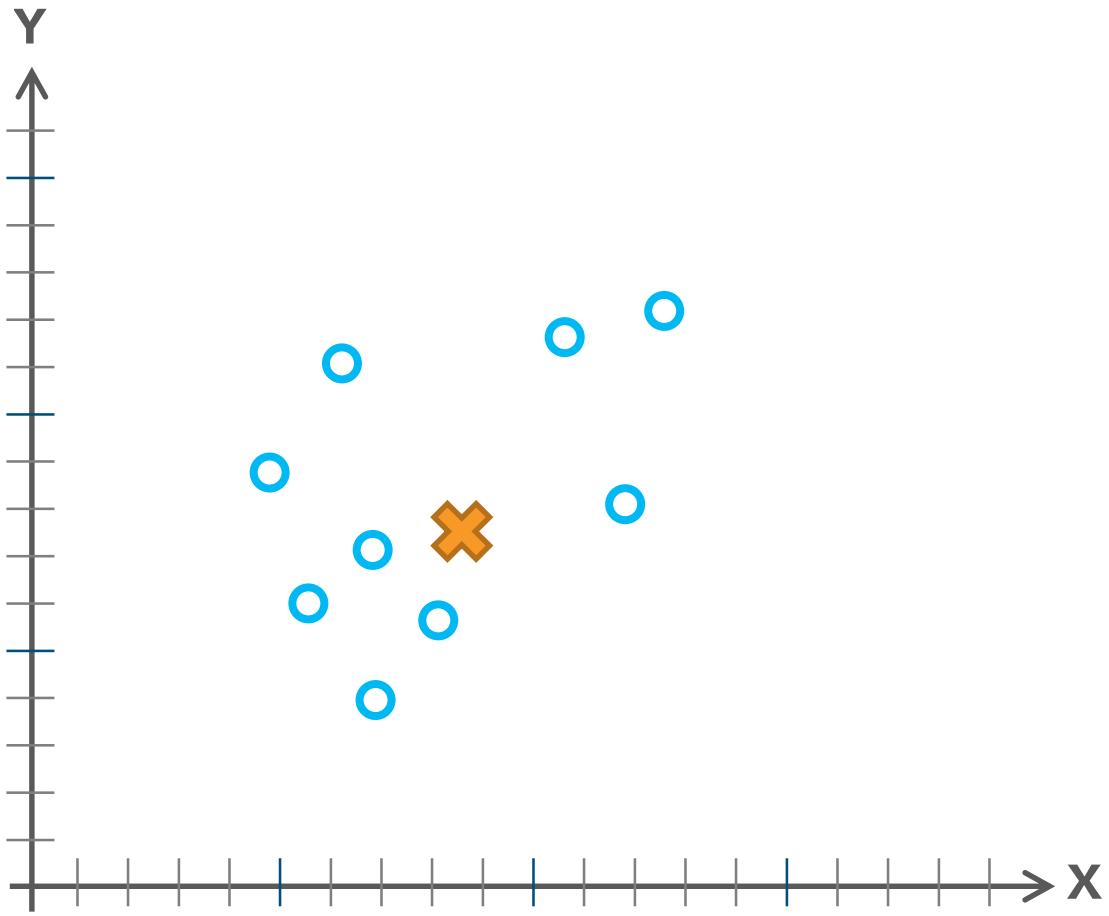
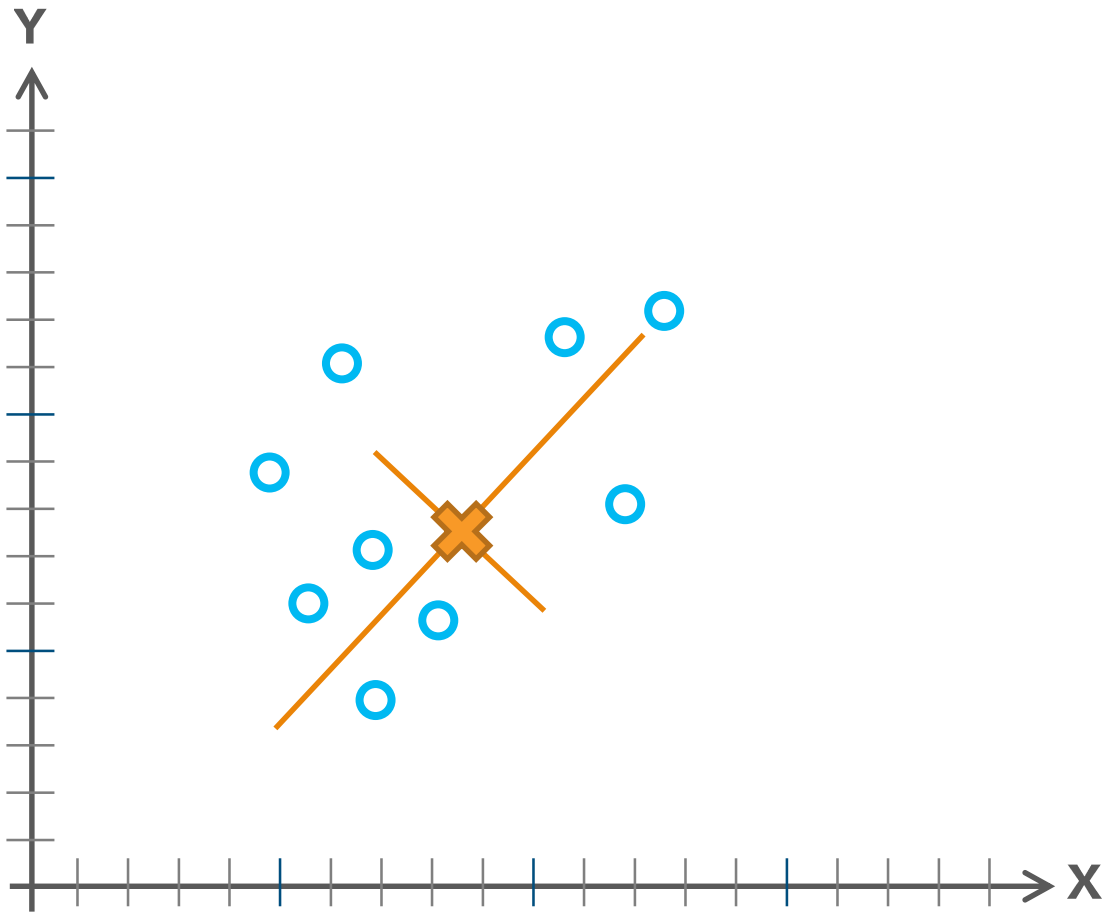

Y
X

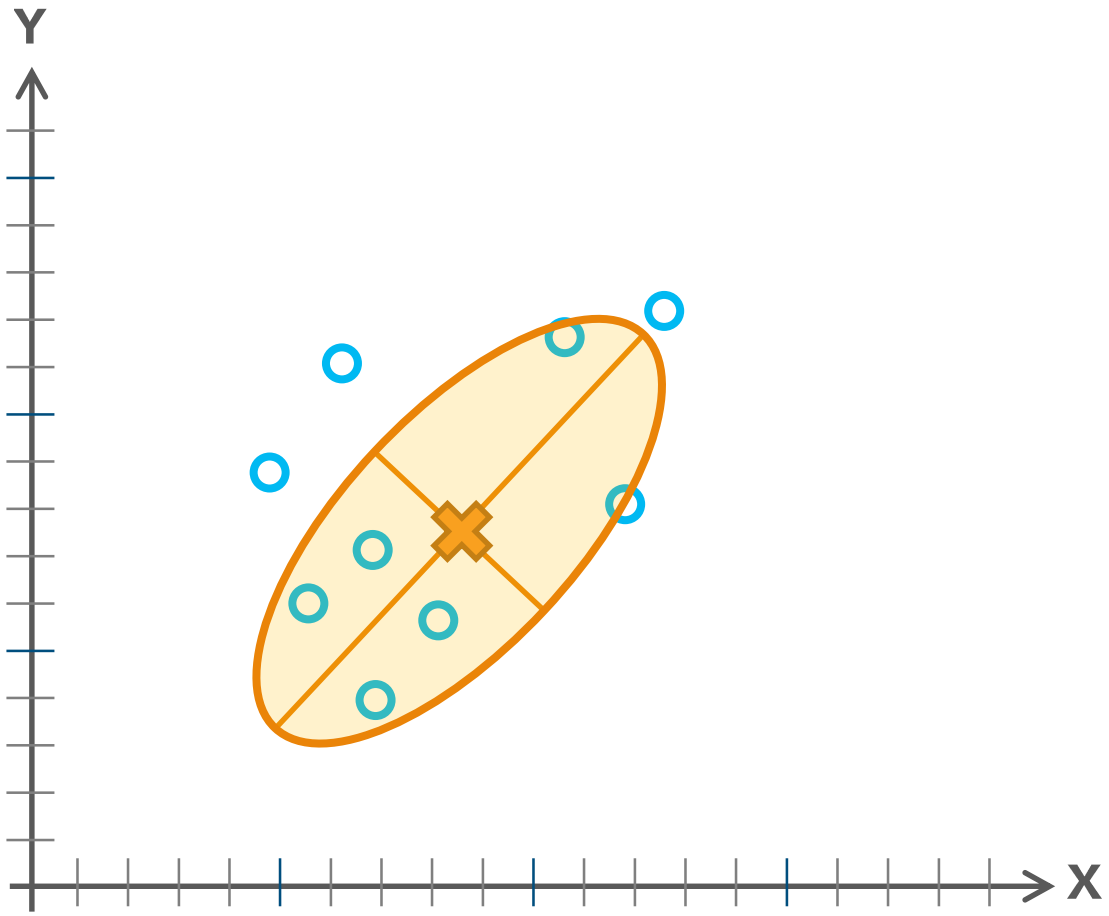
Directional Distribution
(Standard Deviational Ellipse)

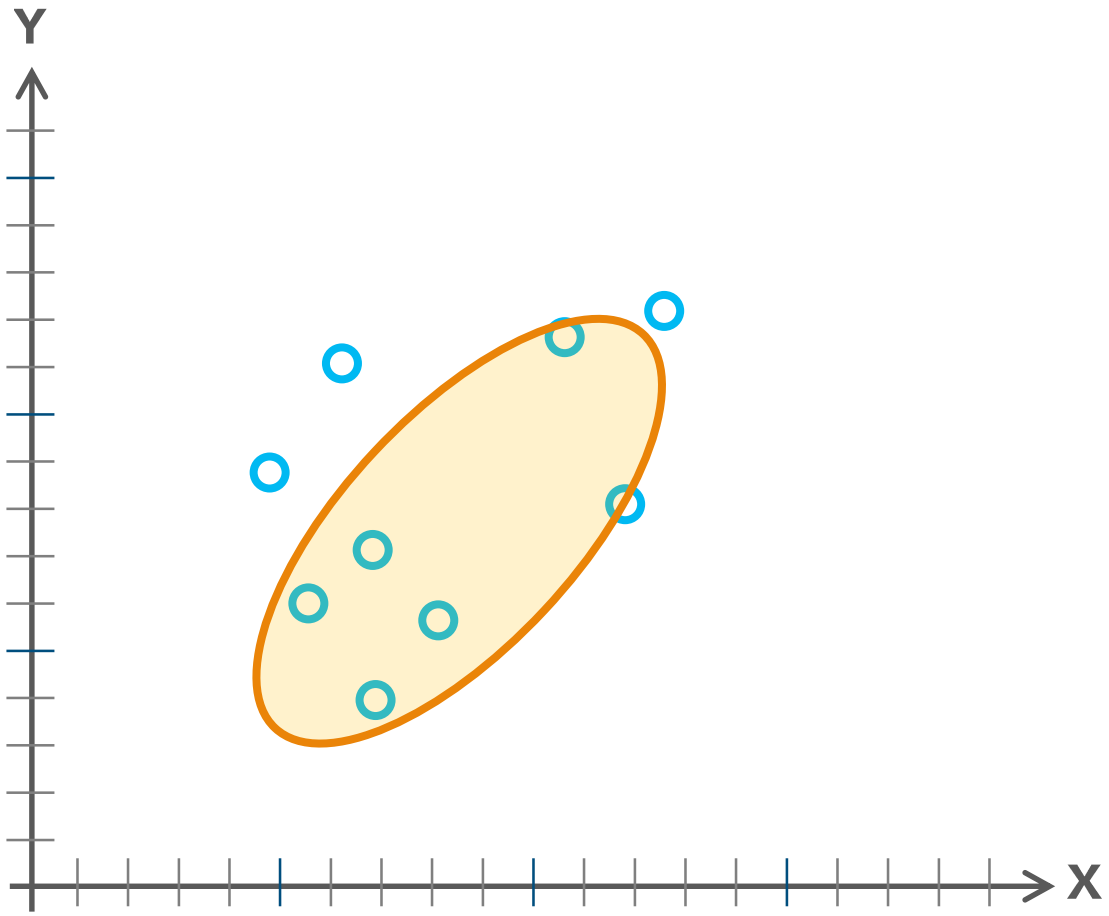
creates standard deviational ellipses to summarize the spatial characteristics of geographic features: central tendency, dispersion, and directional trends


Y
X
mean
center

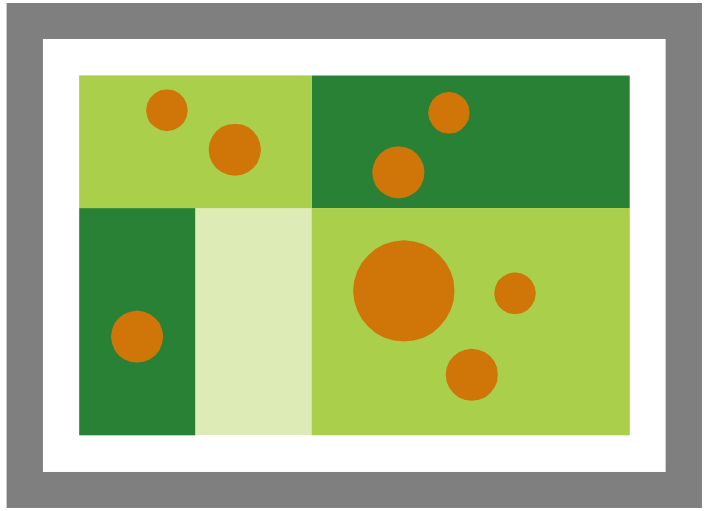

Y
X

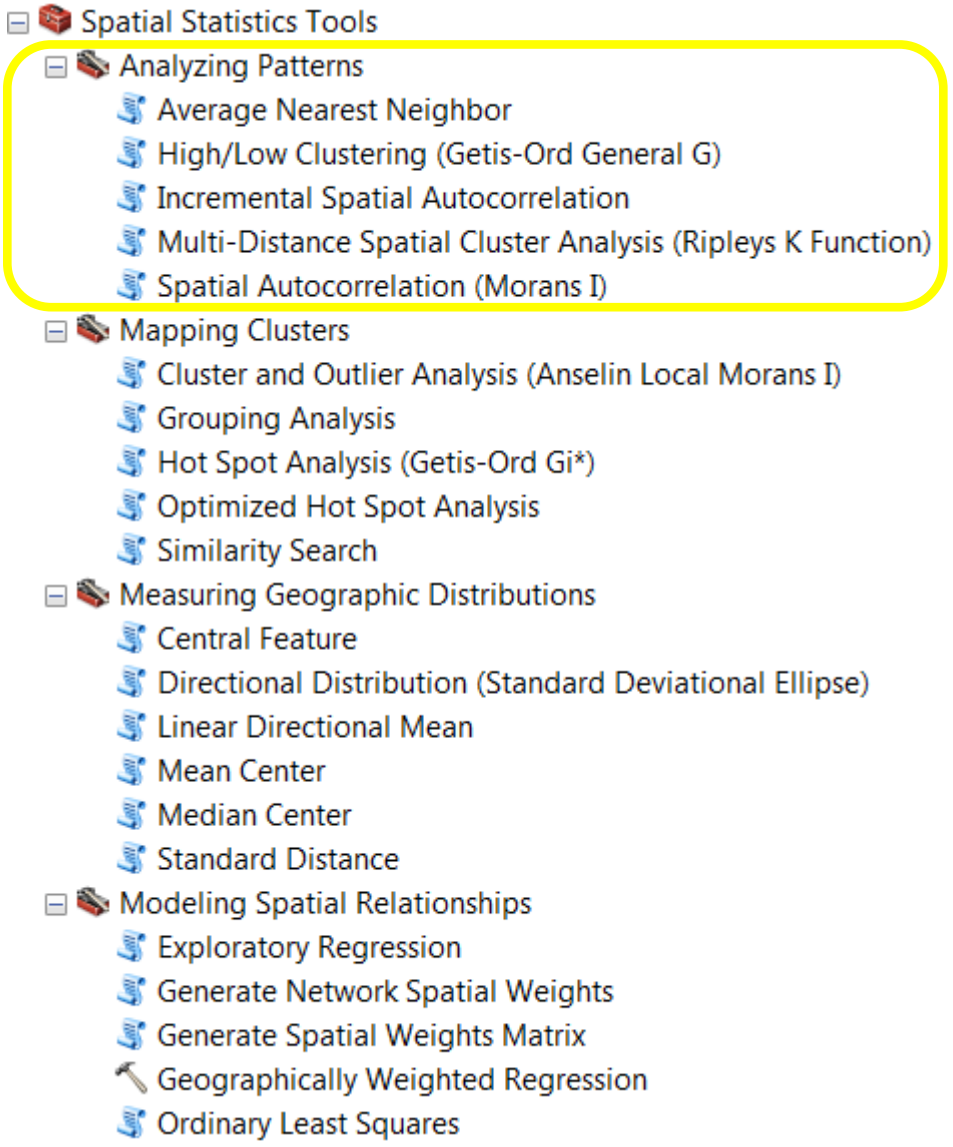

Y
X


Y
X


Y
X

Analyzing Patterns

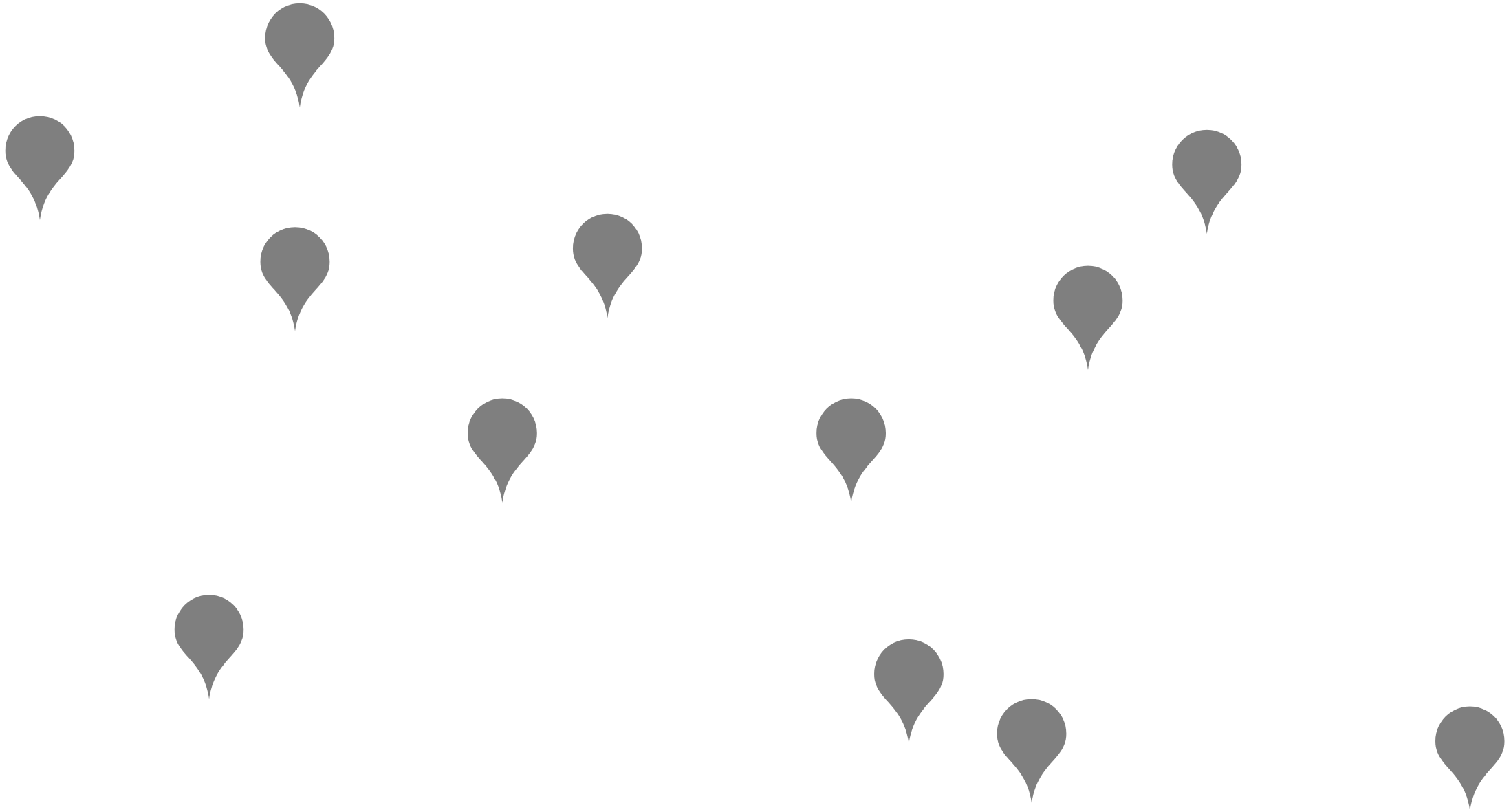


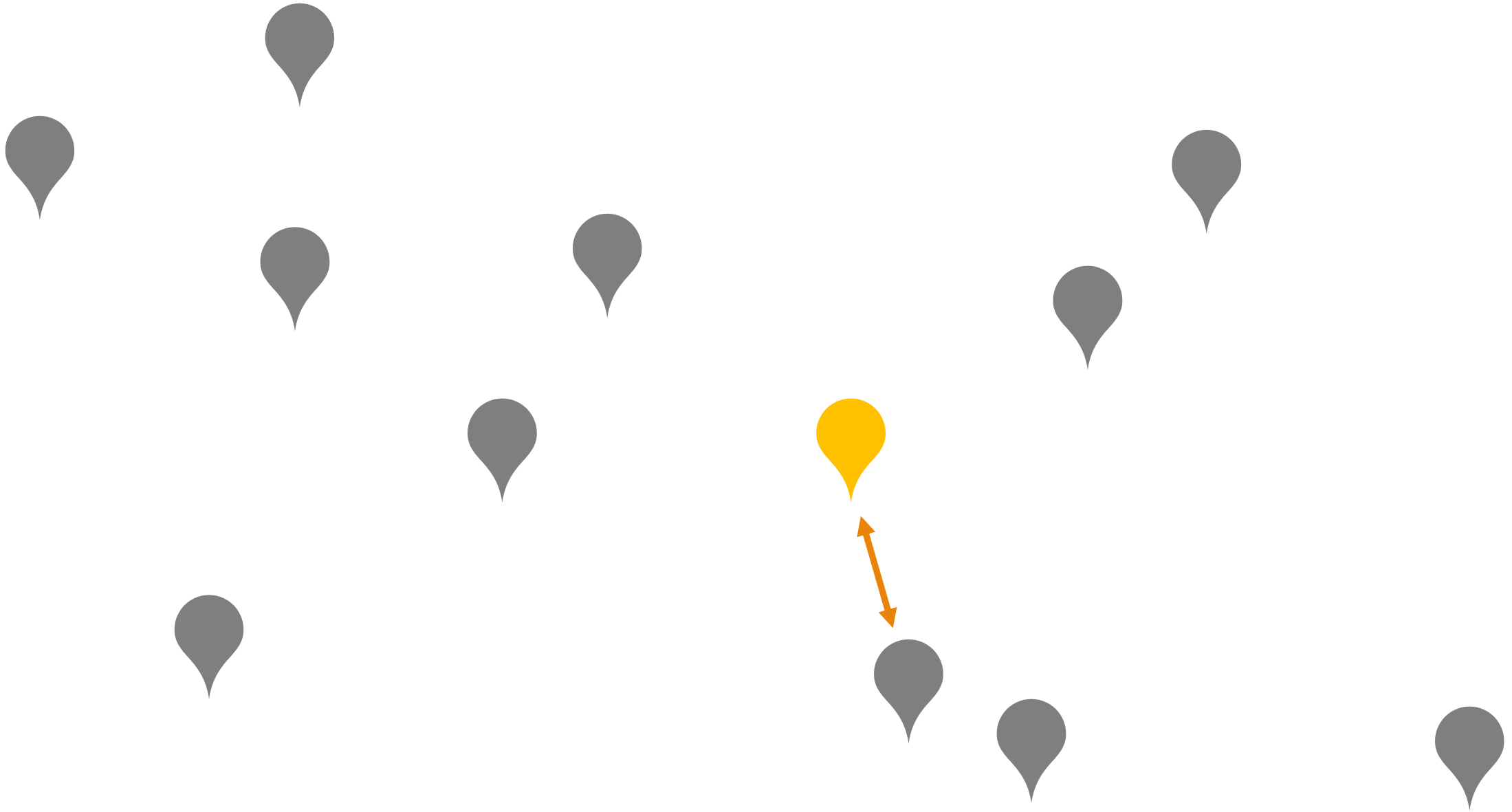


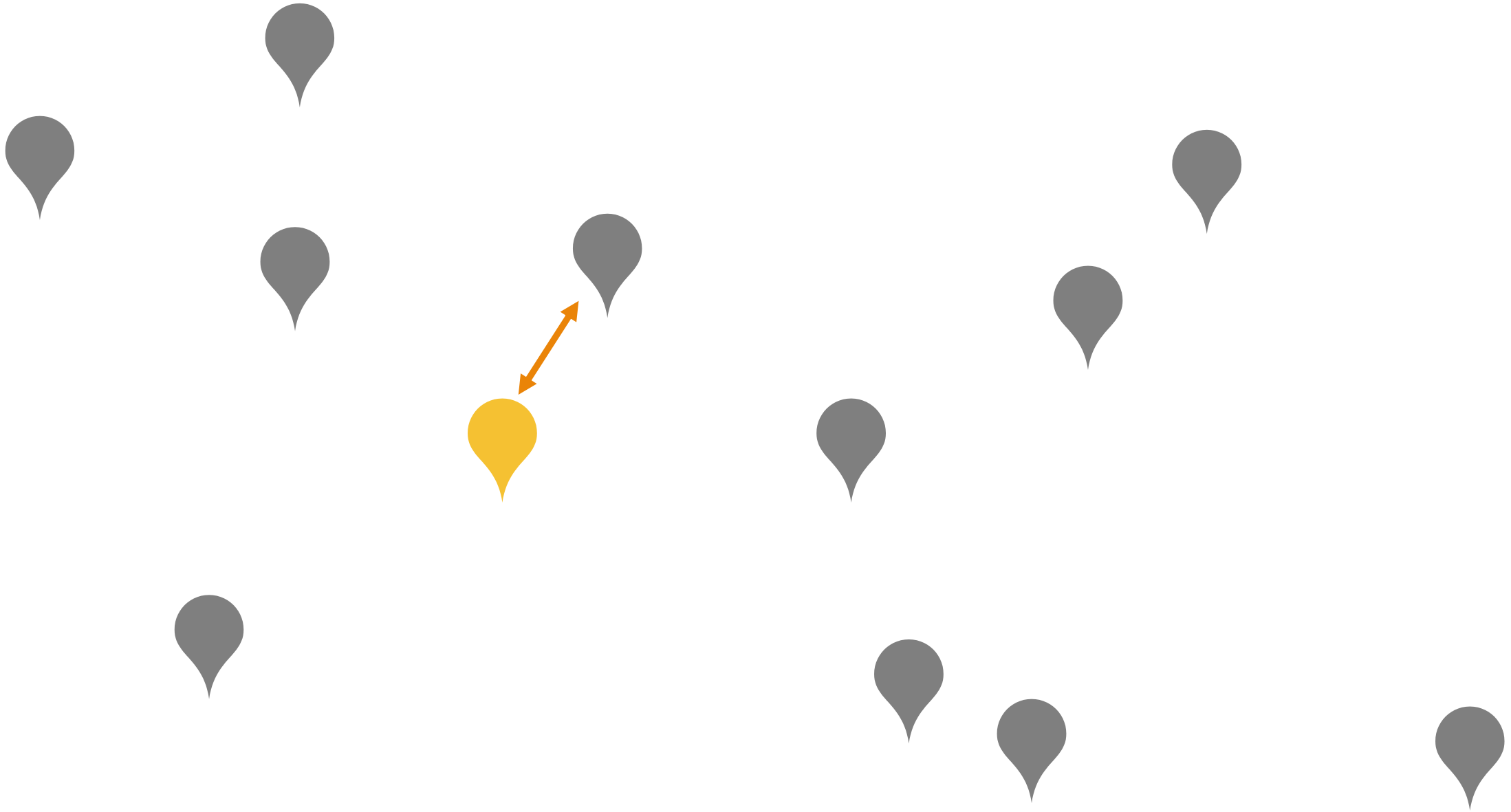
These tools evaluate if features, or the values associated with features, form a clustered, dispersed, or random spatial pattern.

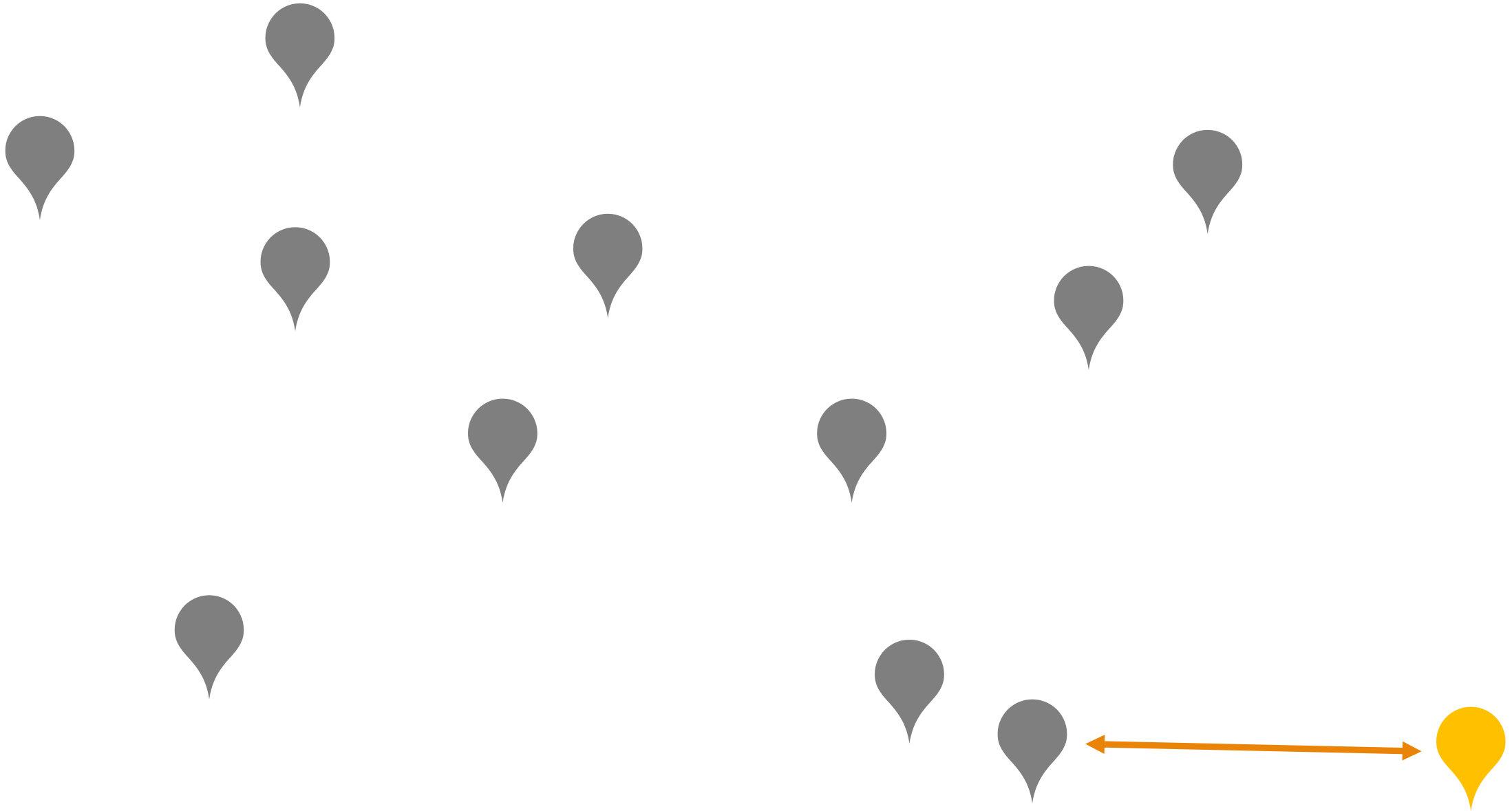
Average Nearest Neighbor

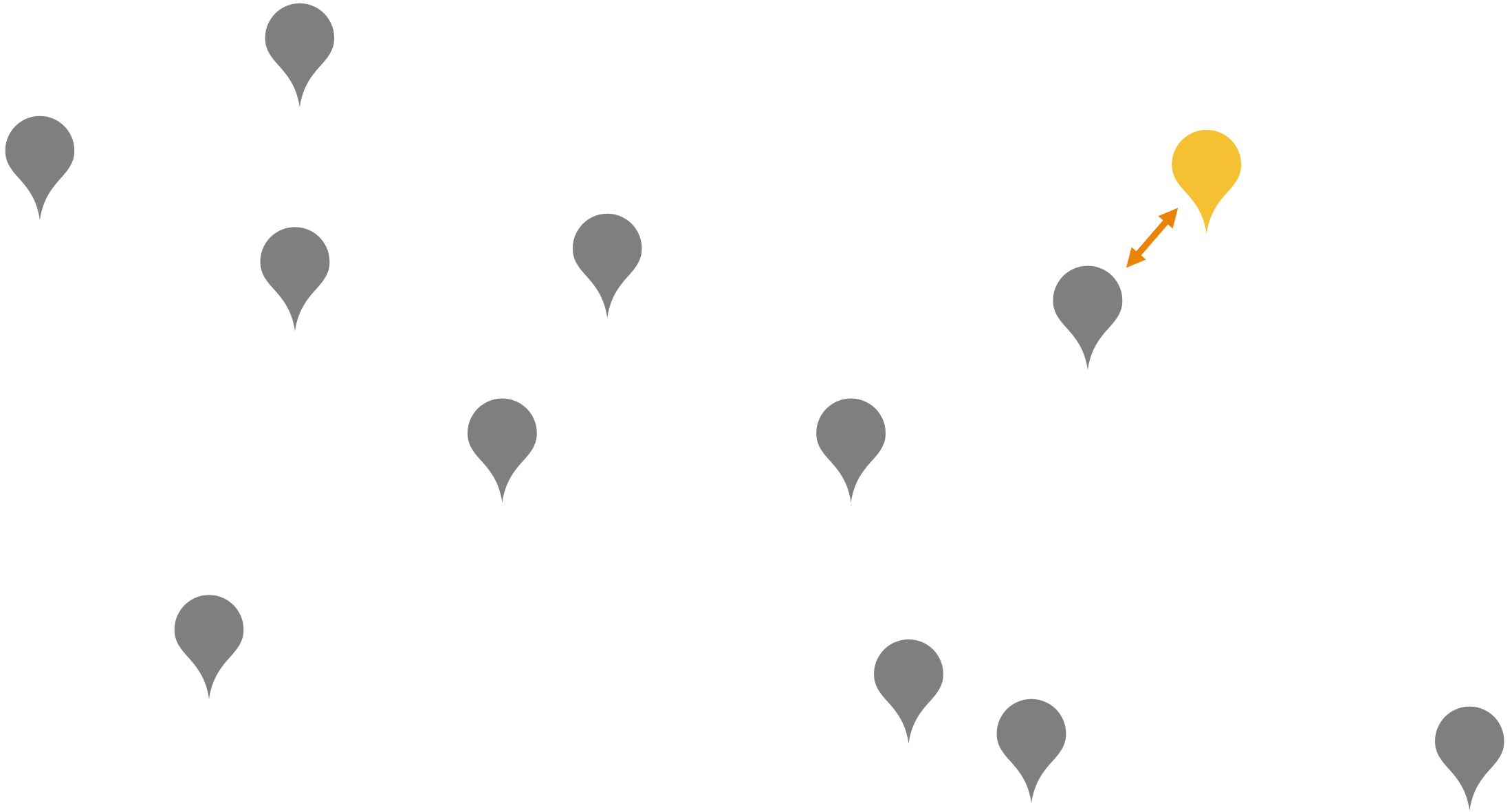
calculates a nearest neighbor index based on the average distance from each feature to its nearest neighboring feature

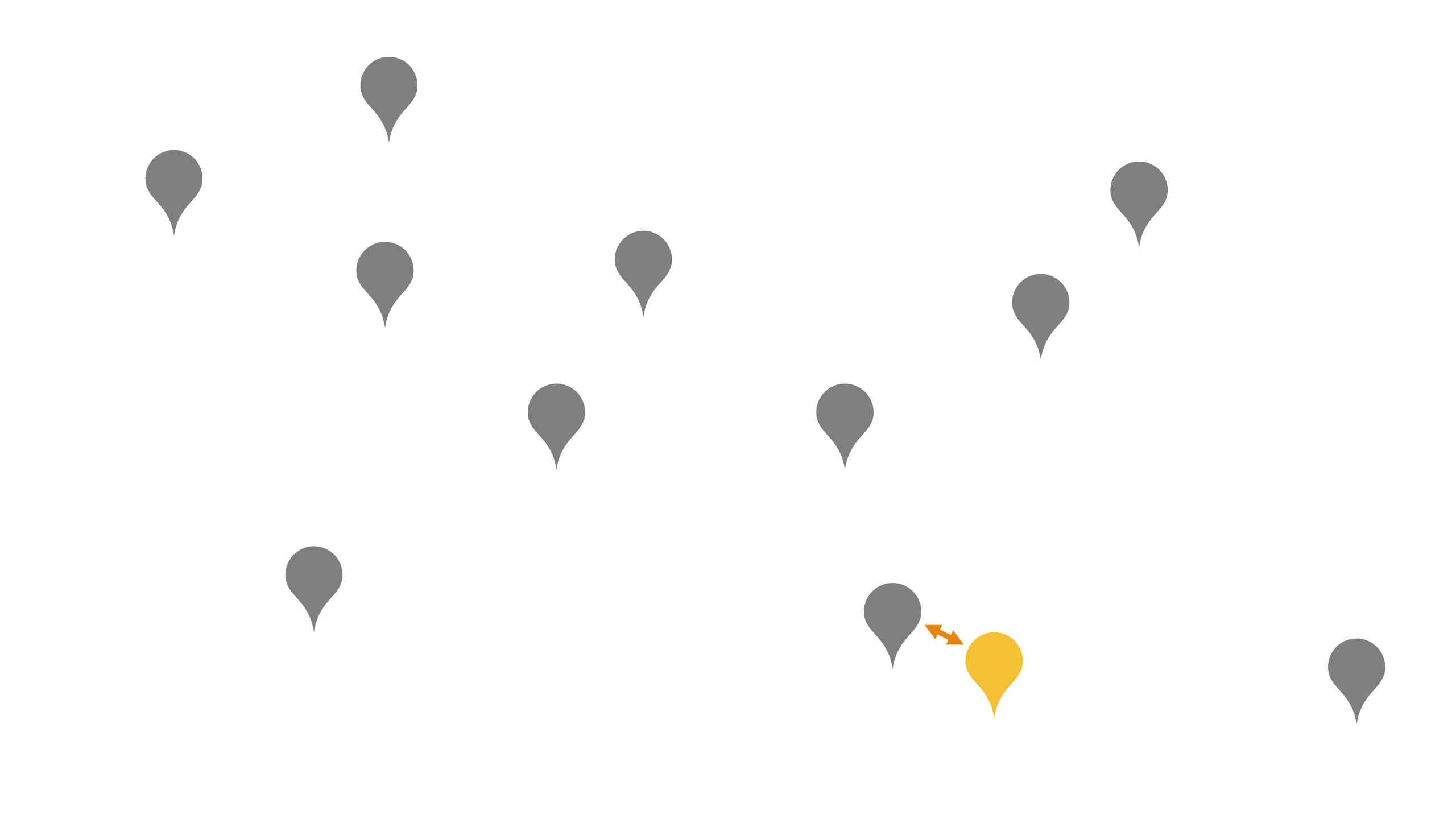


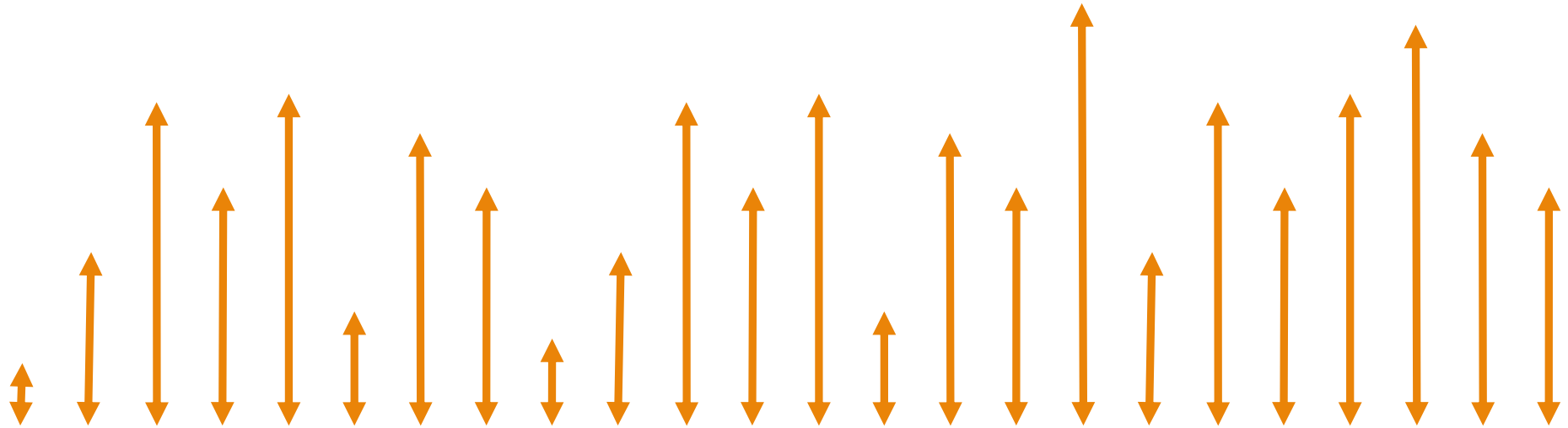








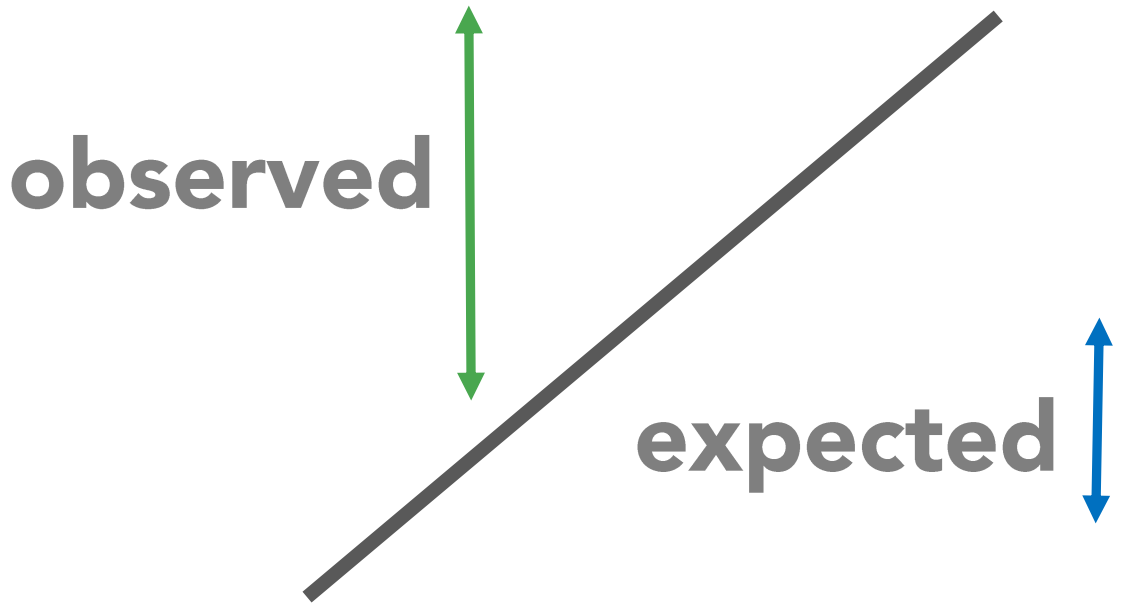




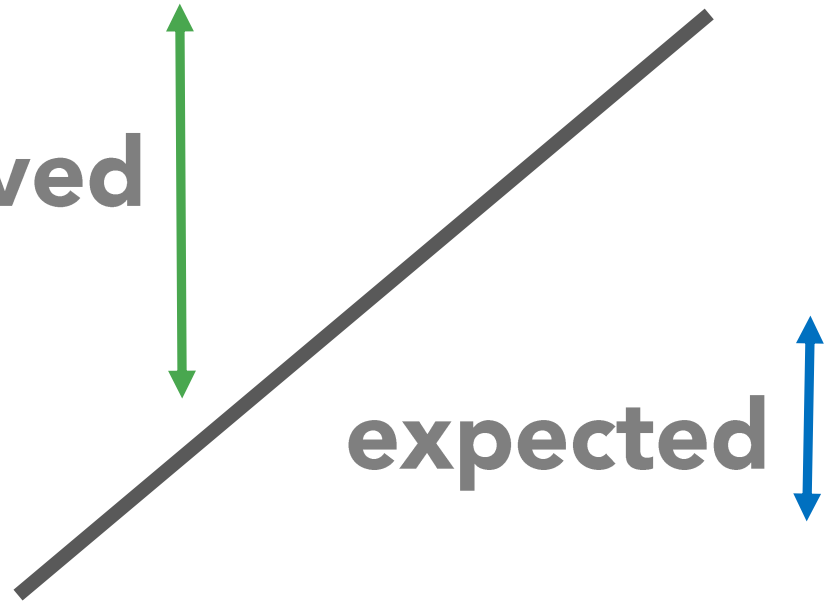
average distance =

expected average distance =

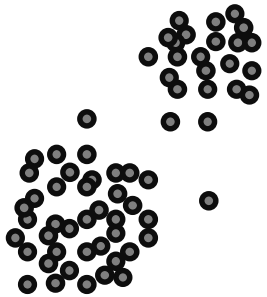
ANN ratio =



ANN ratio =



Clustered



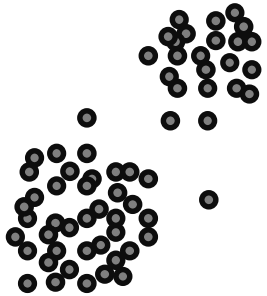
< 1

ANN ratio =

observed

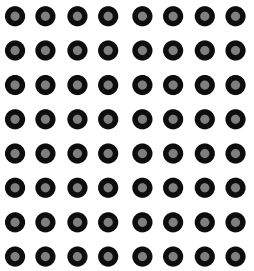
expected

Clustered



< 1 <

Dispersed

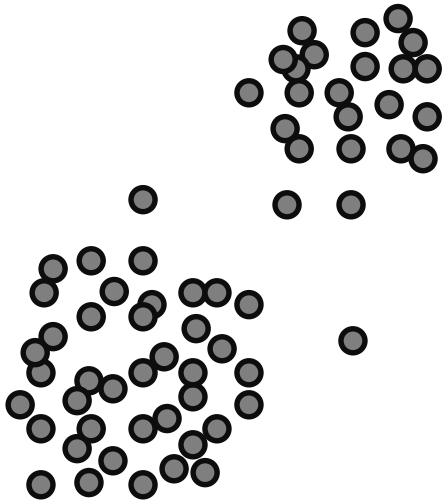


Spatial Autocorrelation
(Moran's I)

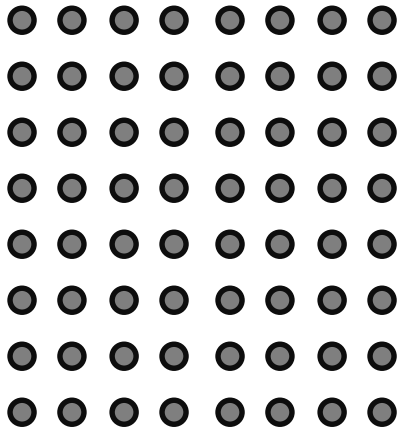
measures spatial autocorrelation based on feature locations and attribute values using the Global Moran's I statistic

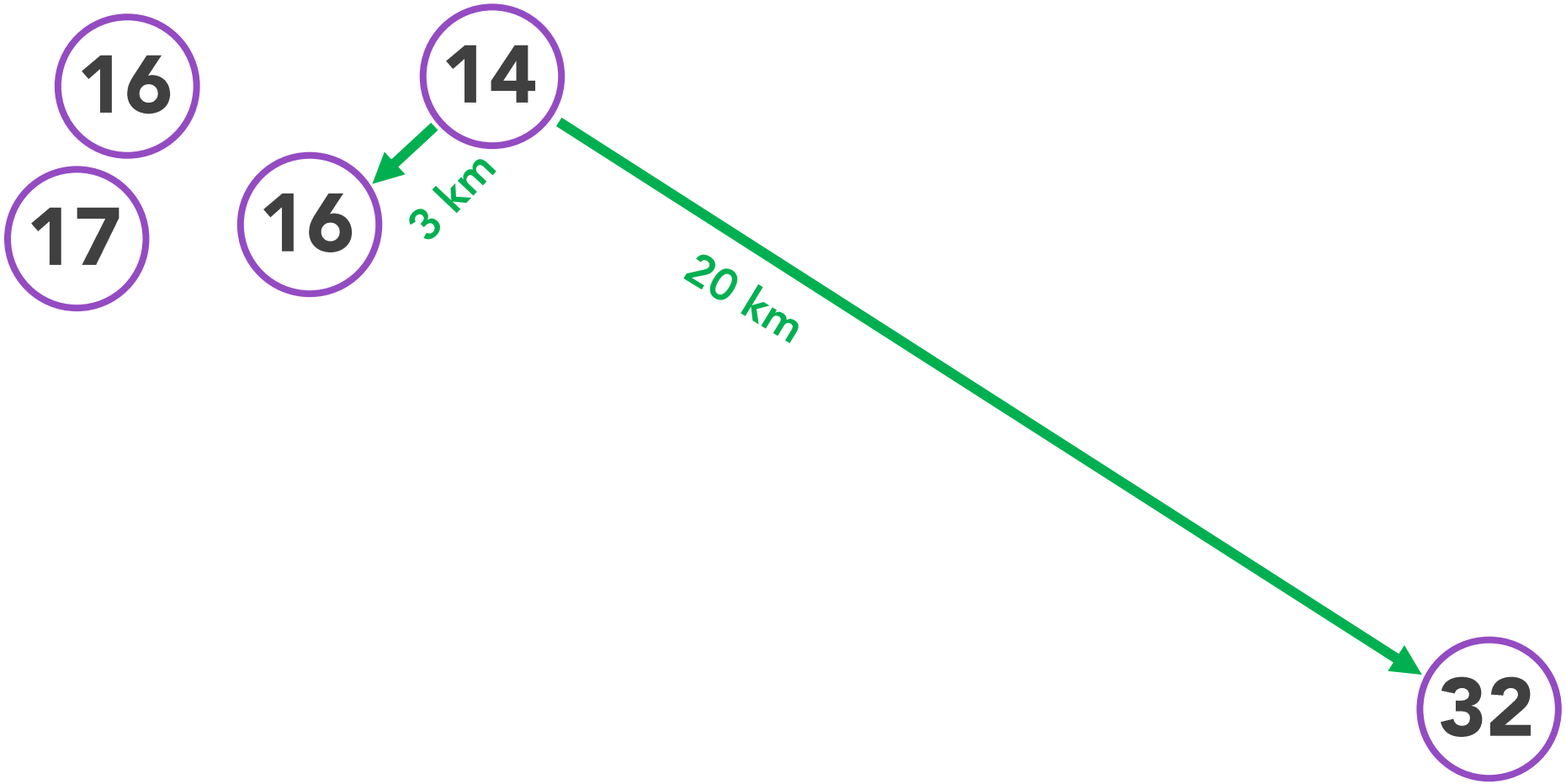
Are distances and values correlated?

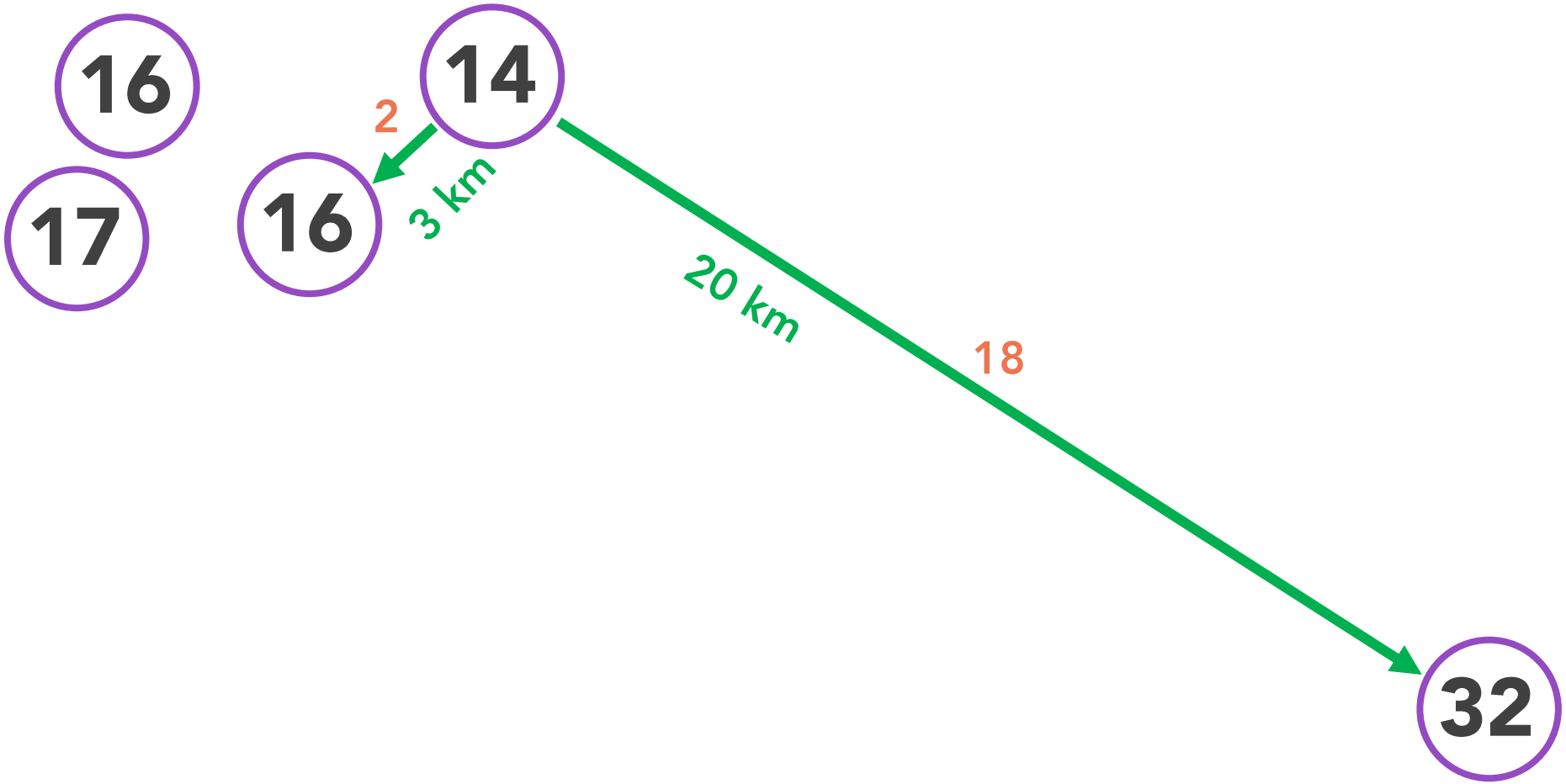
Clustered



Dispersed



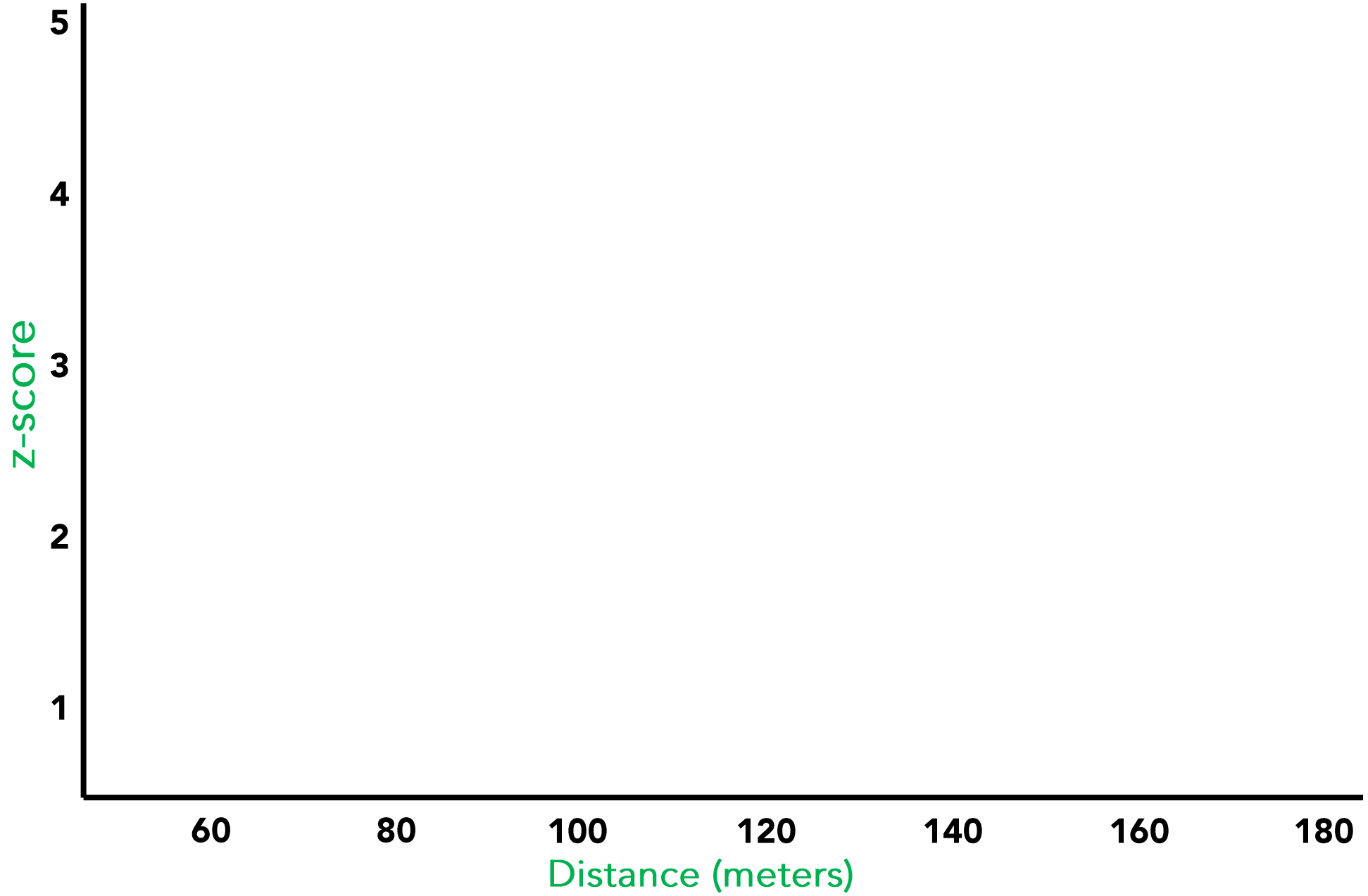

16
17
14
16
3 km
20 km
32


16
17
14
2
16
3 km
20 km
18
32

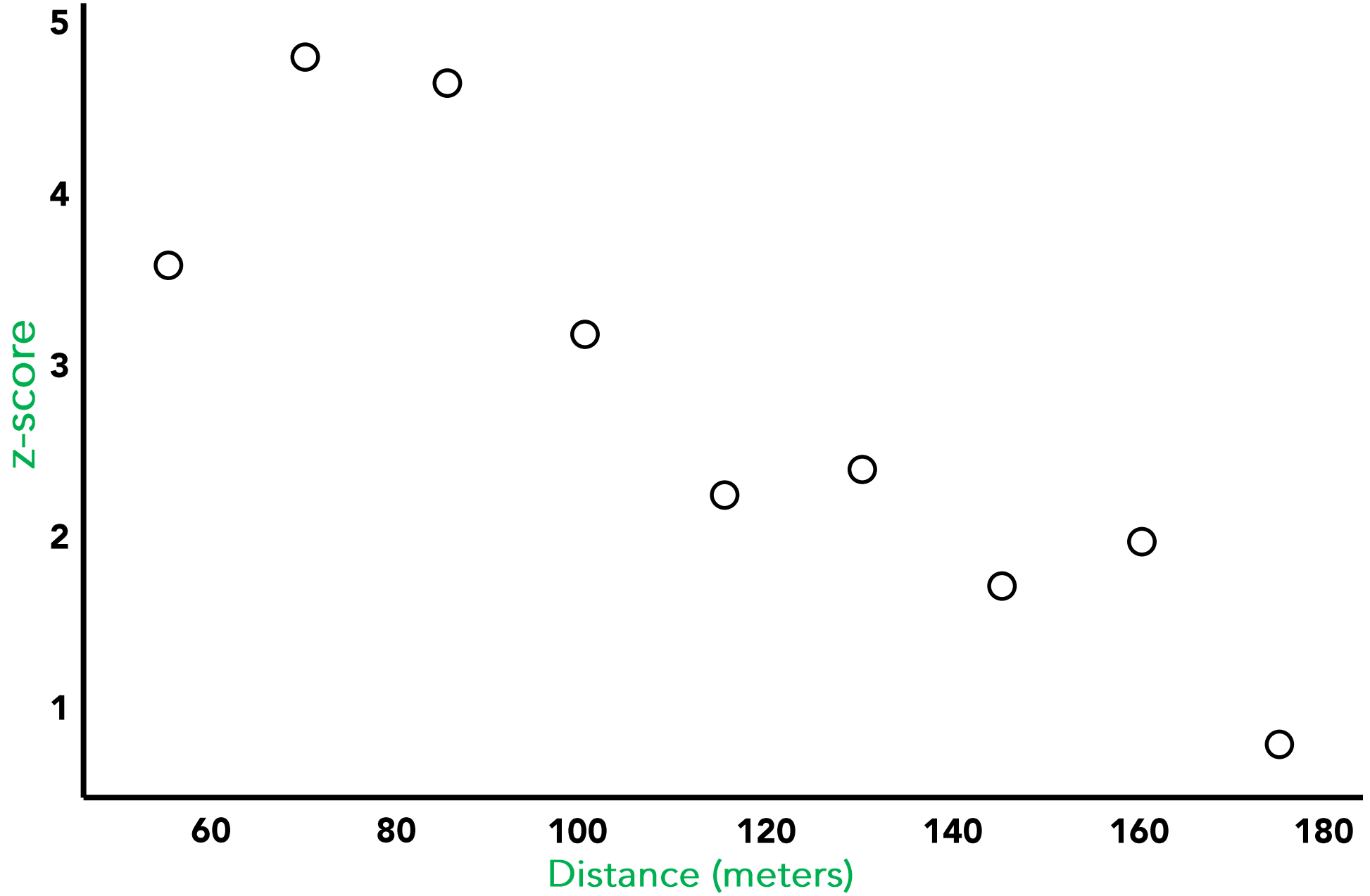
Incremental Spatial Autocorrelation

measures spatial autocorrelation for a series of distances and optionally creates a line graph of those distances and their corresponding z-scores

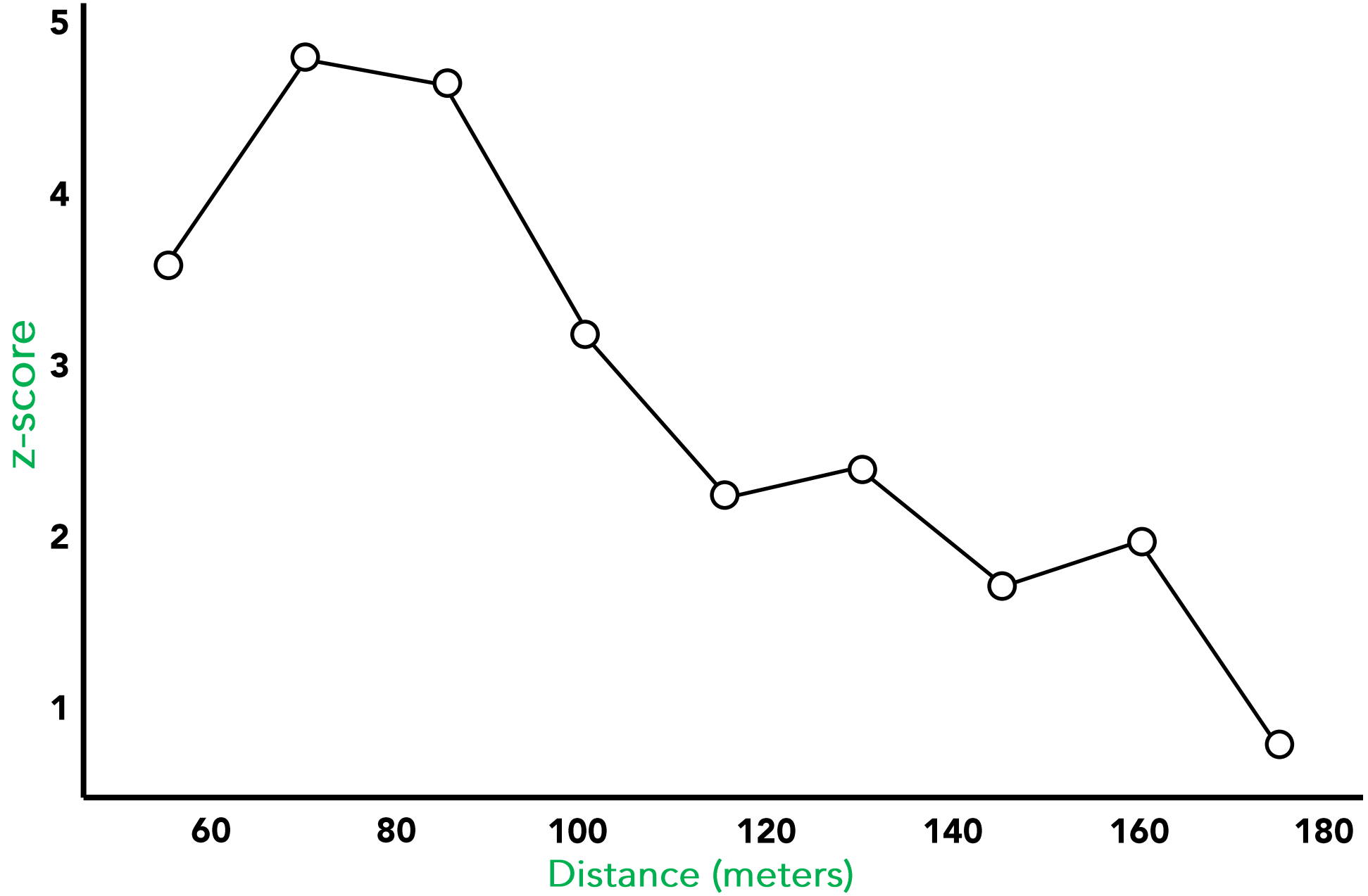
Spatial Autocorrelation by Distance



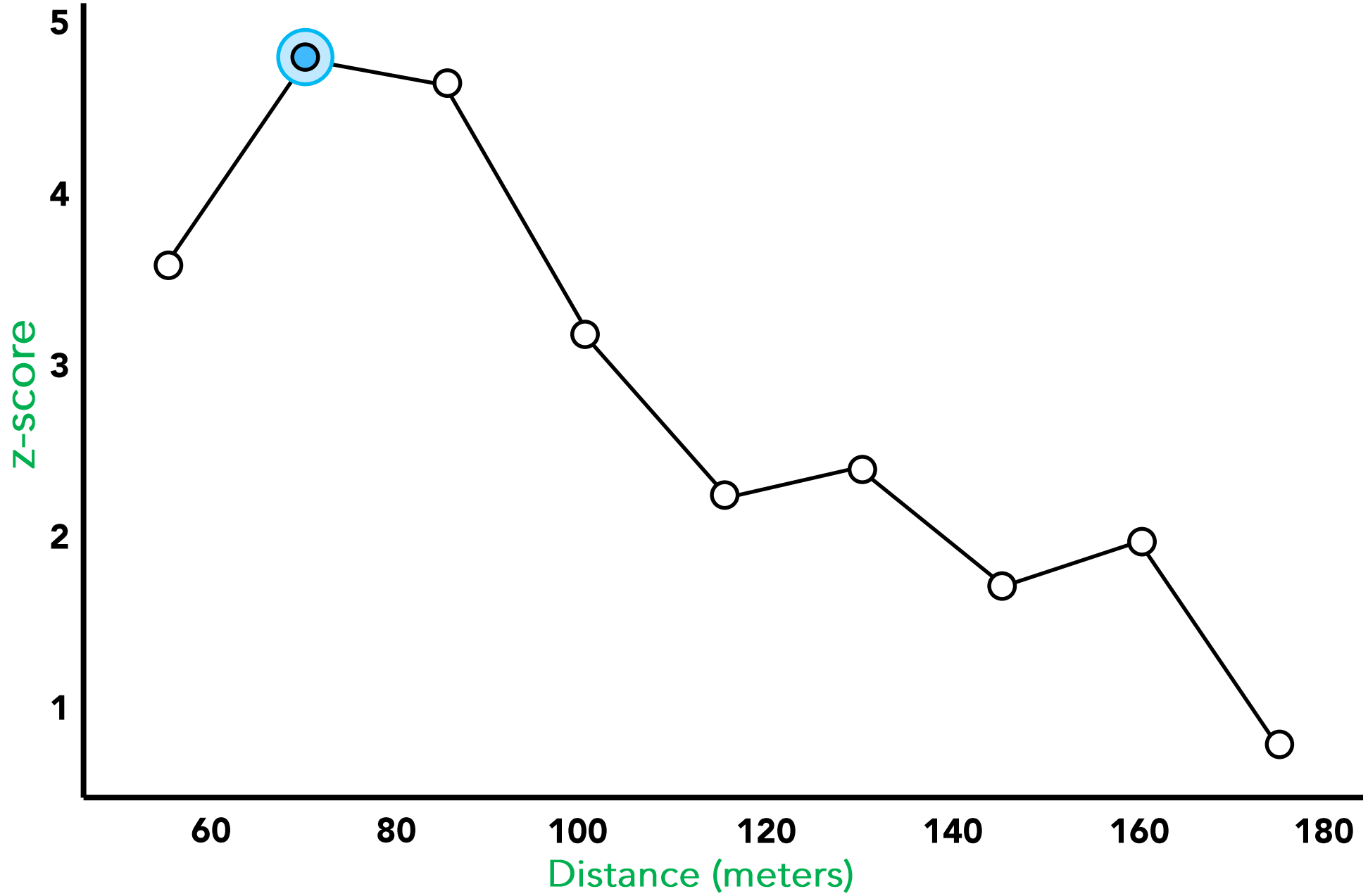
Spatial Autocorrelation by Distance



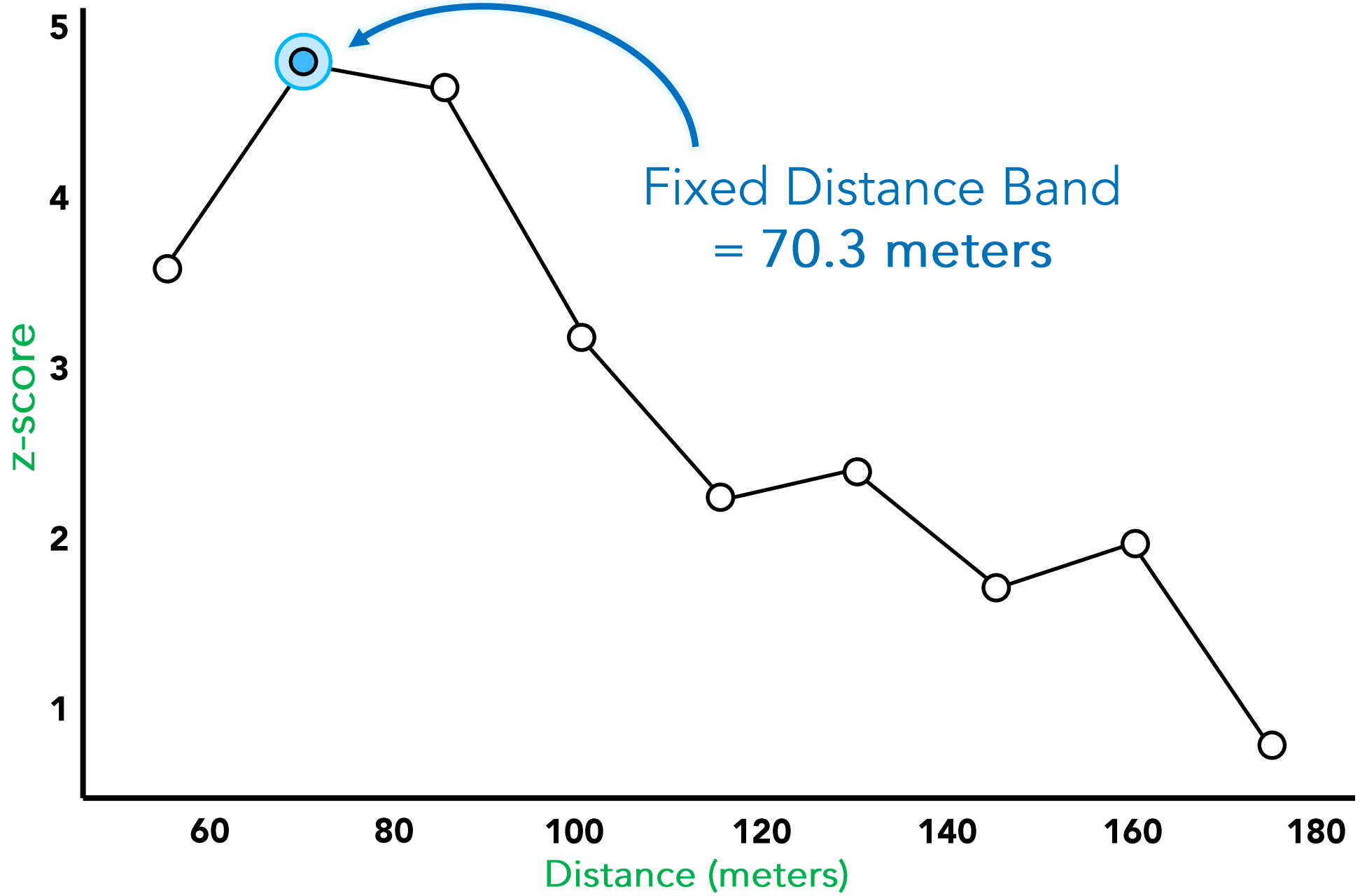
Spatial Autocorrelation by Distance



Spatial Autocorrelation by Distance



Spatial Autocorrelation by Distance

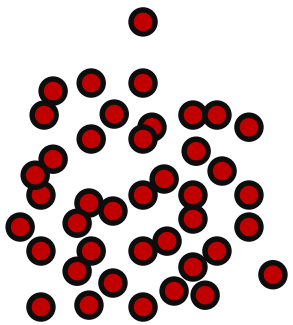


High/Low Clustering
(Getis-Ord General G)

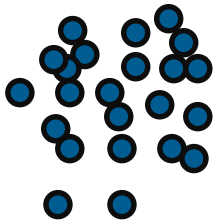
measures the concentration of high or low values for a given study area

What type of clustering is present in the data?

High value clusters



Low value clusters

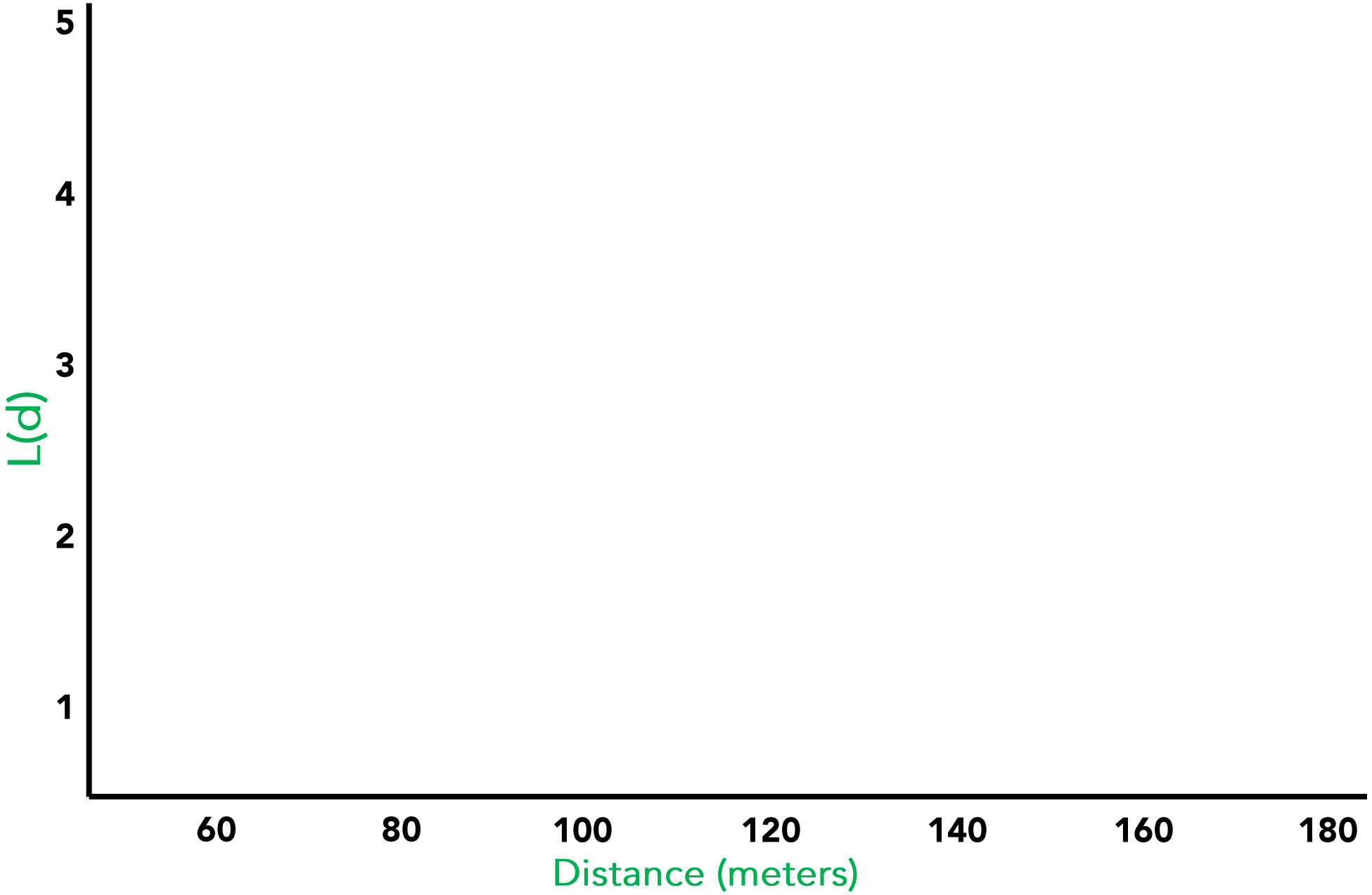


Multi-Distance Spatial Cluster Analysis

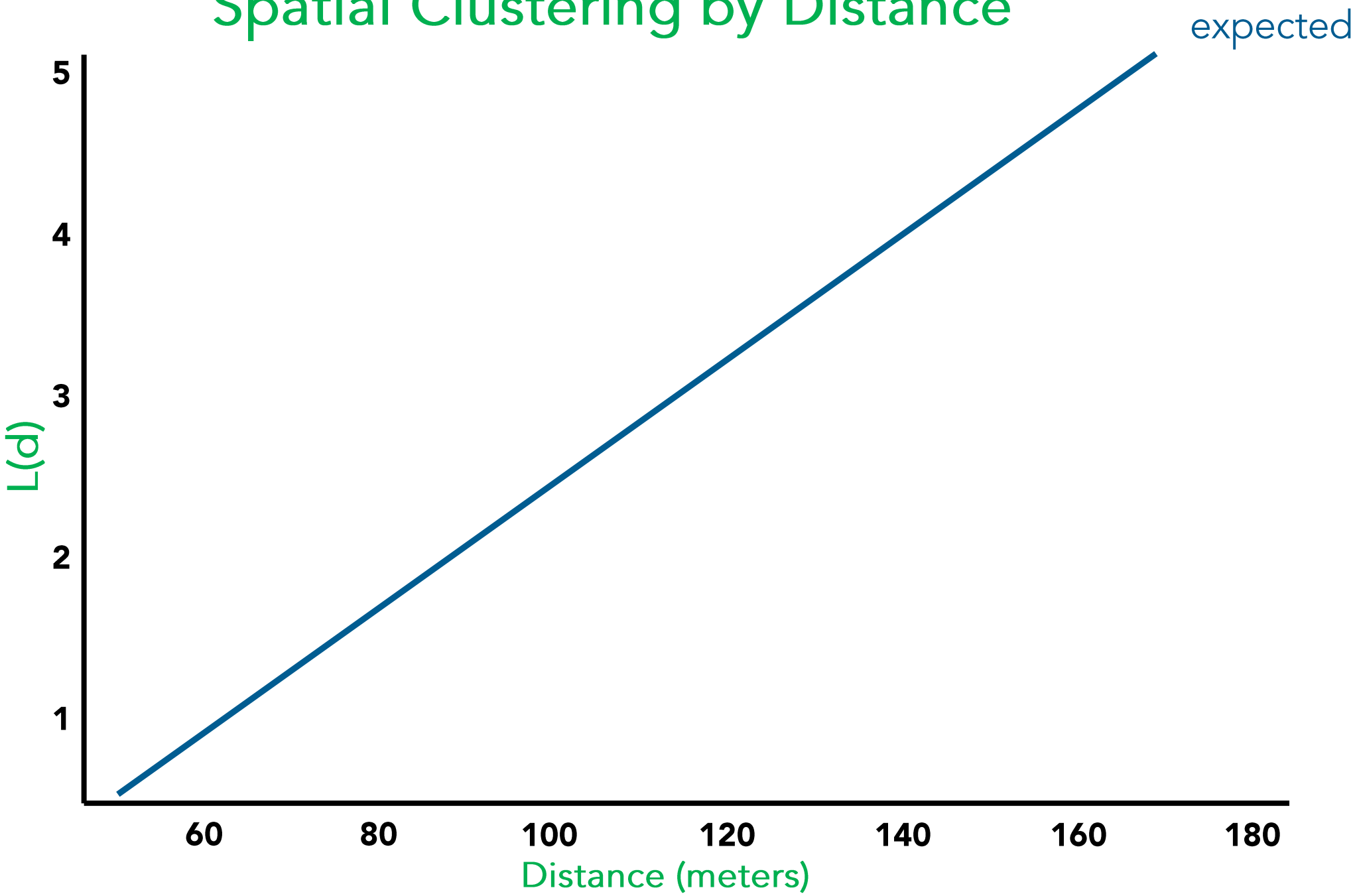
(Ripleys K Function)

determines whether features, or the values associated with features, exhibit statistically significant clustering or dispersion over a range of distances

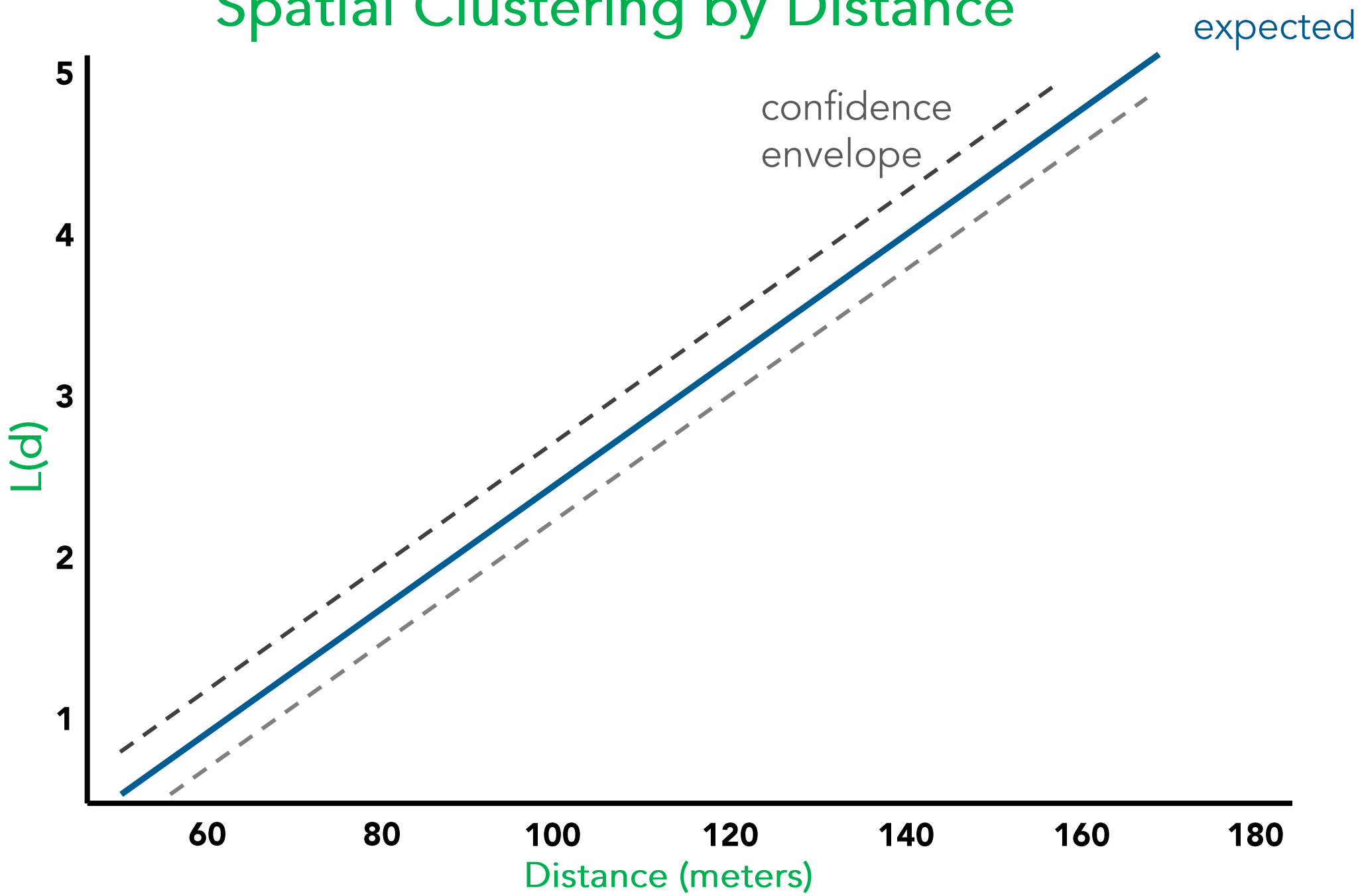
Spatial Clustering by Distance



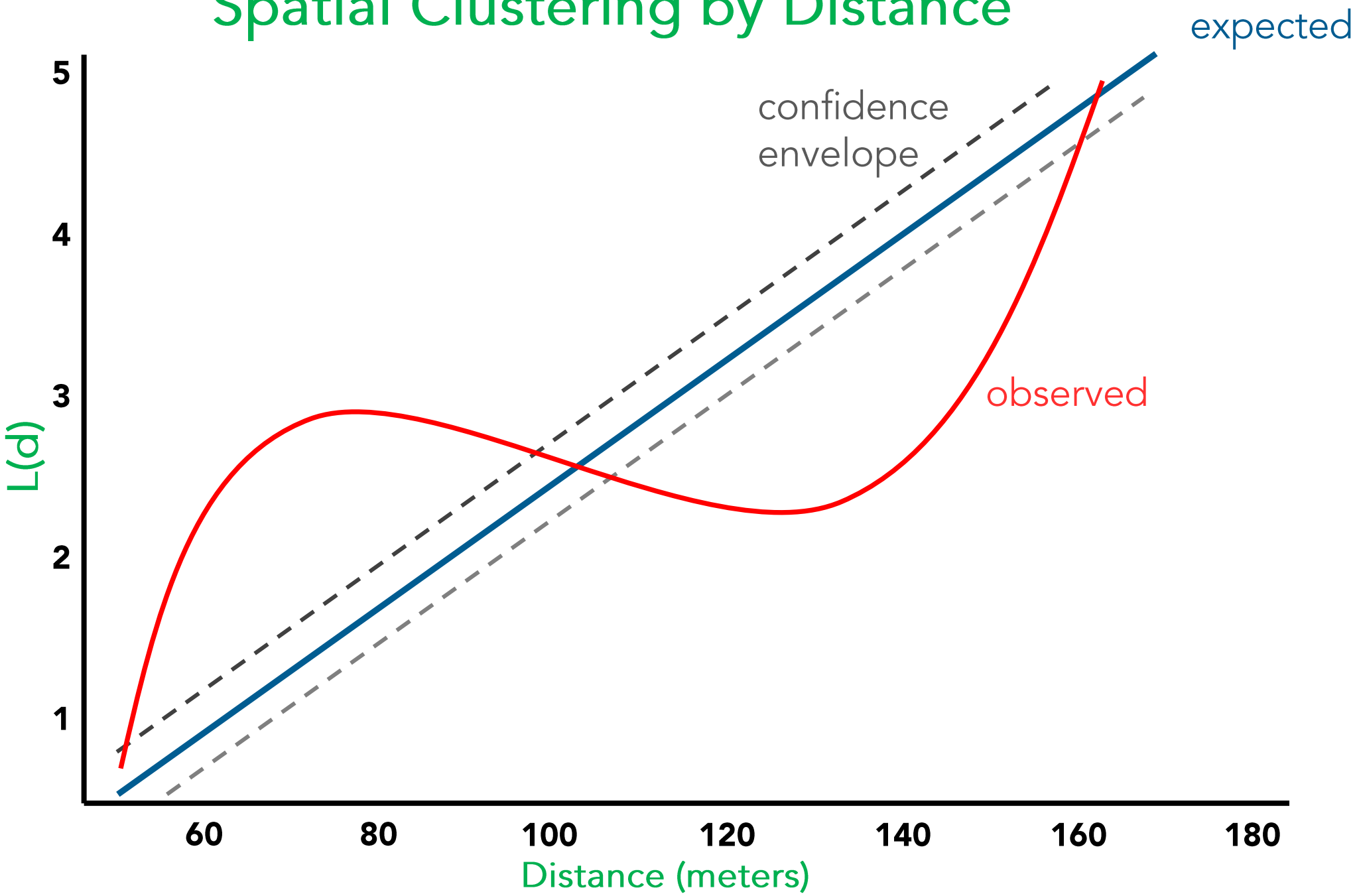
Spatial Clustering by Distance



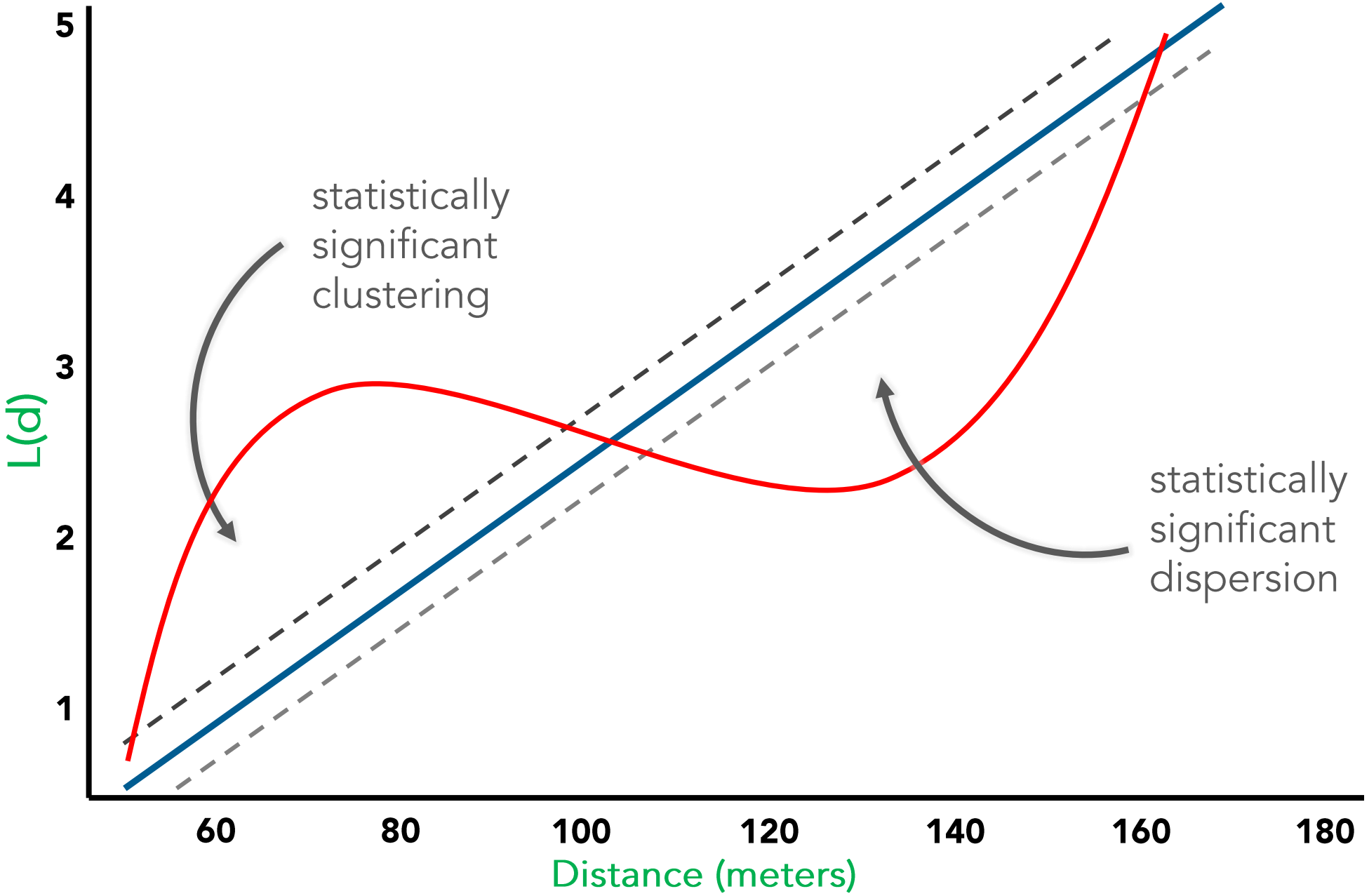
Spatial Clustering by Distance



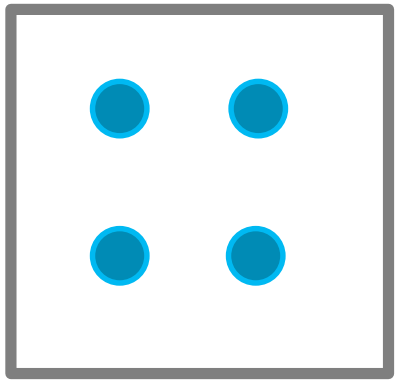
Spatial Clustering by Distance



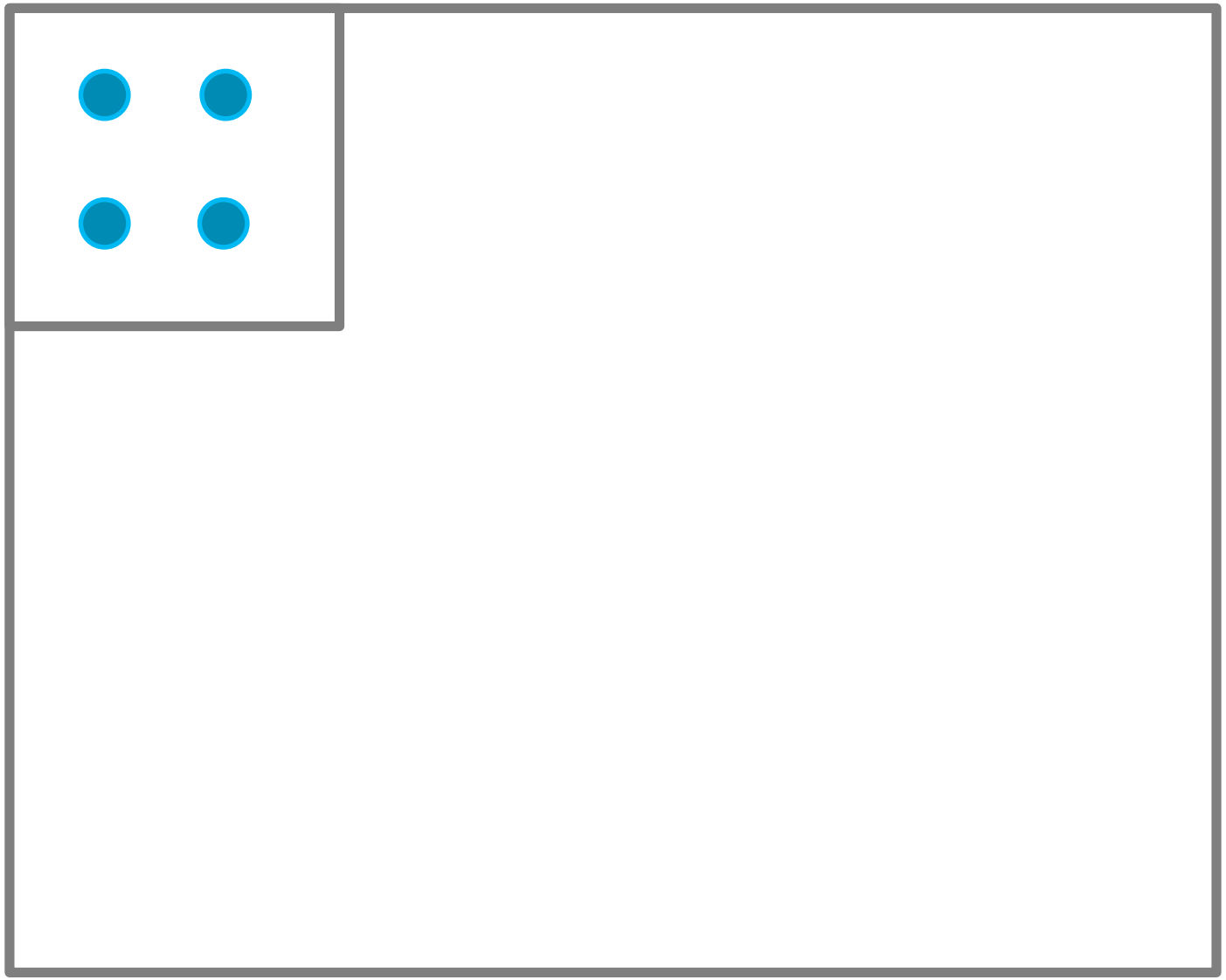
Spatial Clustering by Distance



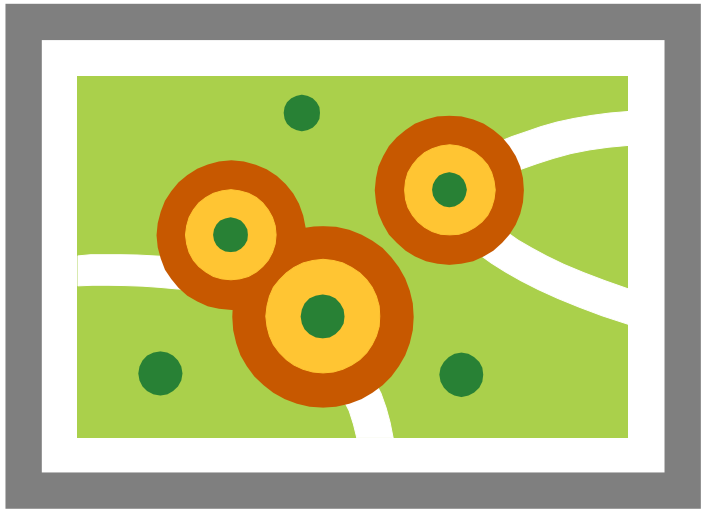
dispersed

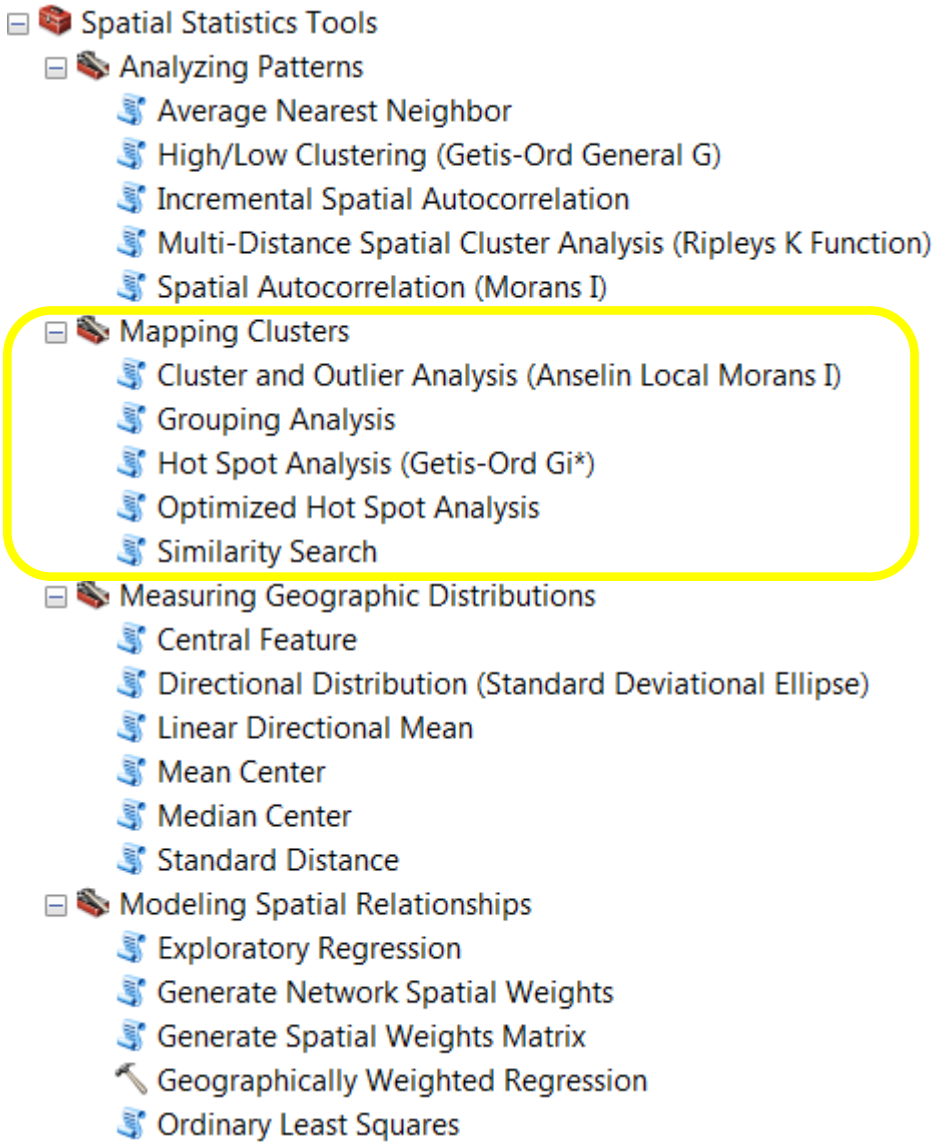


clustered



Mapping Clusters





These tools may be used to identify statistically significant hot spots, cold spots, or spatial outliers. There are also tools to identify or group features with similar characteristics.

Similarity Search

identifies which candidate features are most similar or most dissimilar to one or more input features based on feature attributes

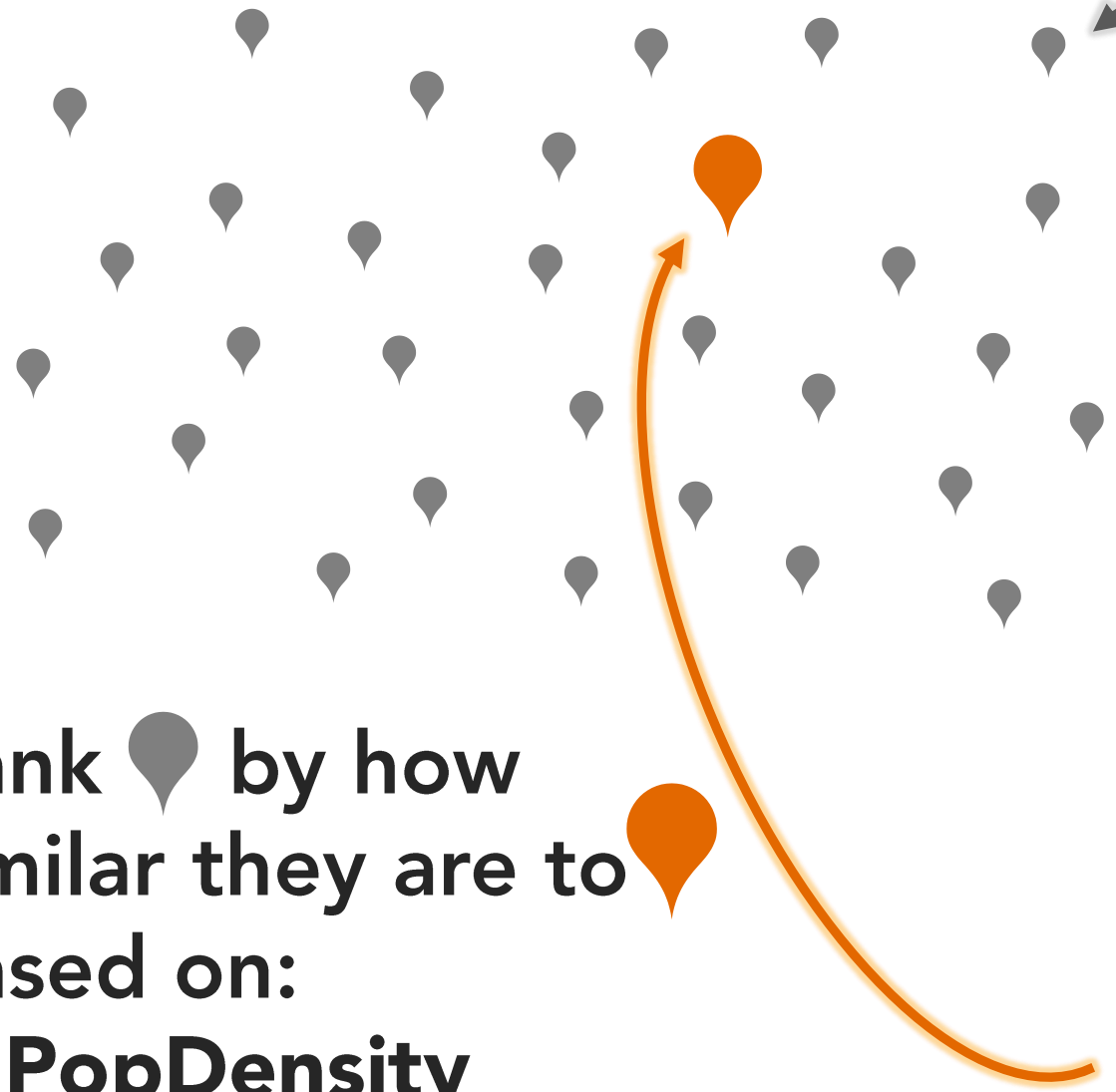
potential
store
locations

high performing store
potential store locations

LocID	PopDensity	AvIncome	DistToCompetition

LocID	PopDensity	AvIncome	DistToCompetition

LocID	PopDensity	AvIncome	DistToCompetition

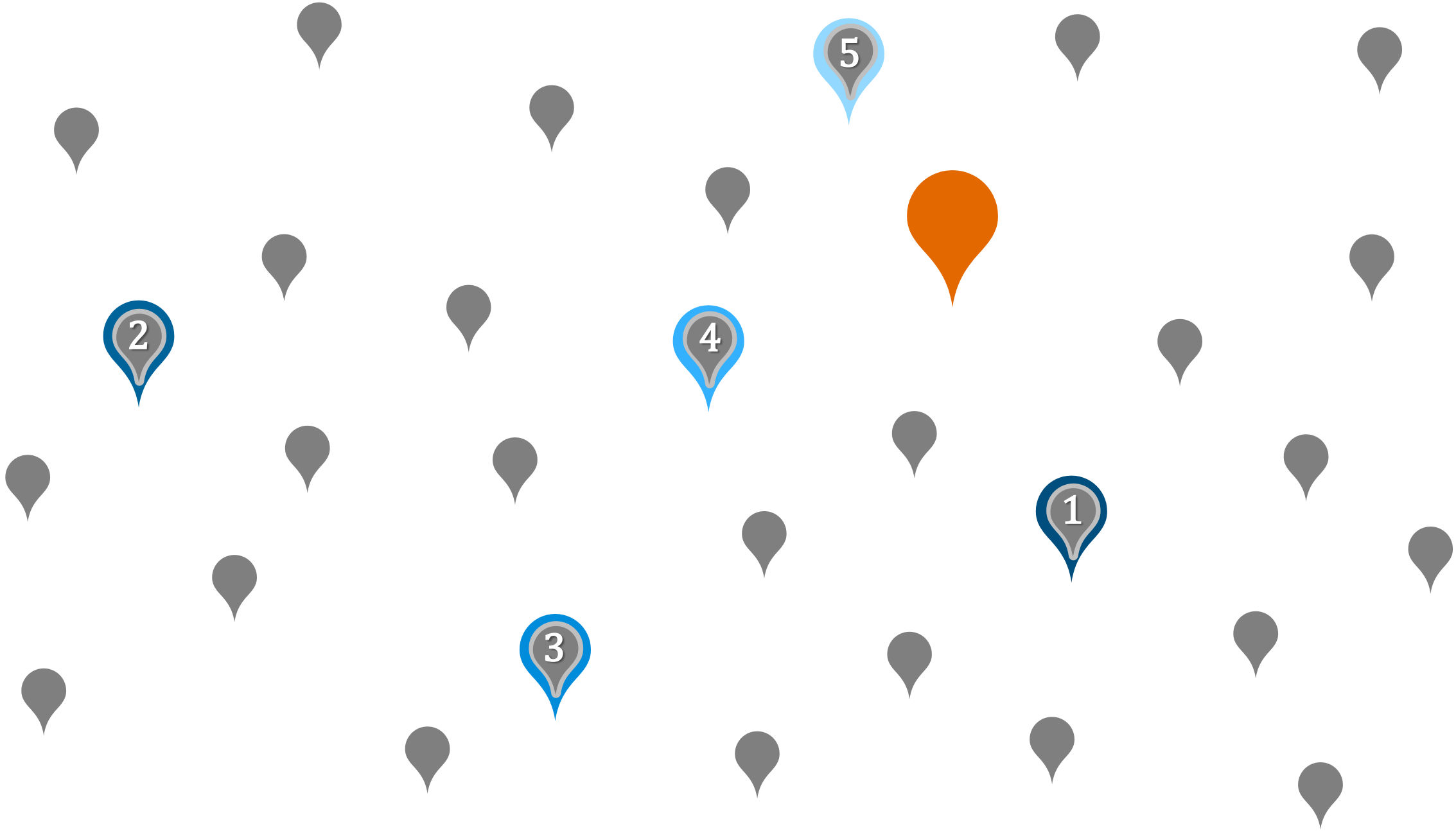


Rank by how similar they are to based on:

- **PopDensity**
- **AvIncome**
- **DistToCompetition**

LocID	PopDensity	AvIncome	DistToCompetition

LocID	PopDensity	AvIncome	DistToCompetition


5
2
4
1
3

3 Match Methods

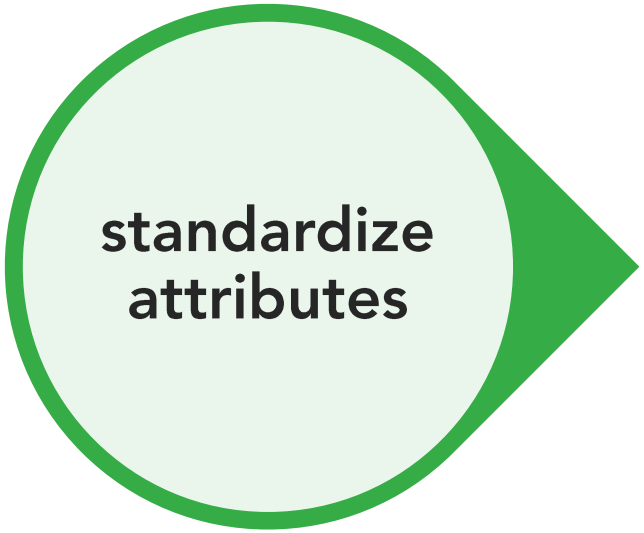
Attribute Values

Ranked Attribute Values

Attribute Profiles

Attribute Values

Attribute Values



Z-transform:

$$(x - \bar{x}) / SD$$

Attribute Values

standardize attributes

Population = 14,159

% Uninsured = .26

Distance (km) = 535.89

Attribute Values

standardize attributes

Population = -.7932

% Uninsured = 3.8462

Distance (km) = .6433

Attribute Values

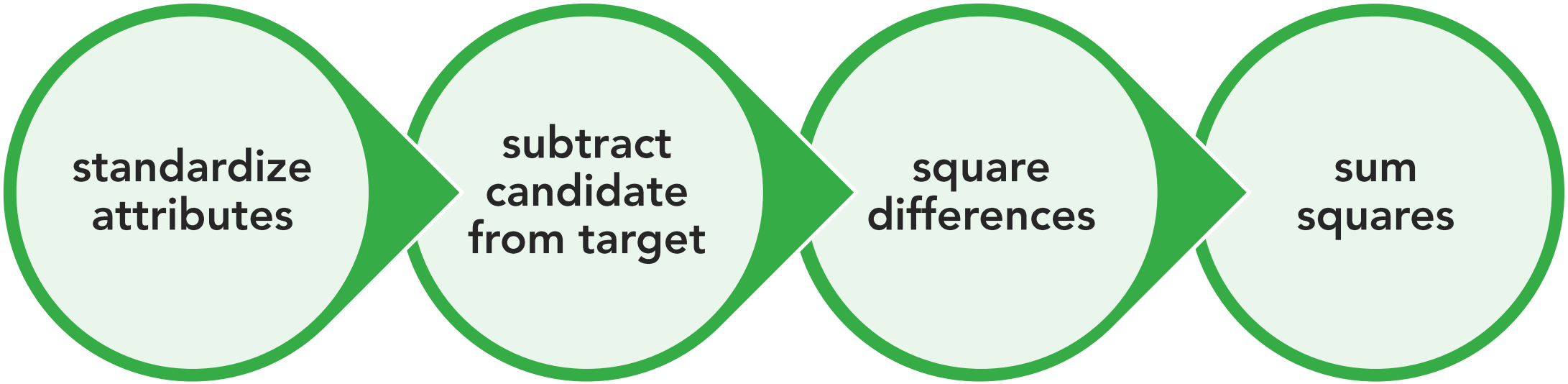
standardize attributes

Population = -.7932

% Uninsured = 3.8462

Distance (km) = .6433

Attribute Values

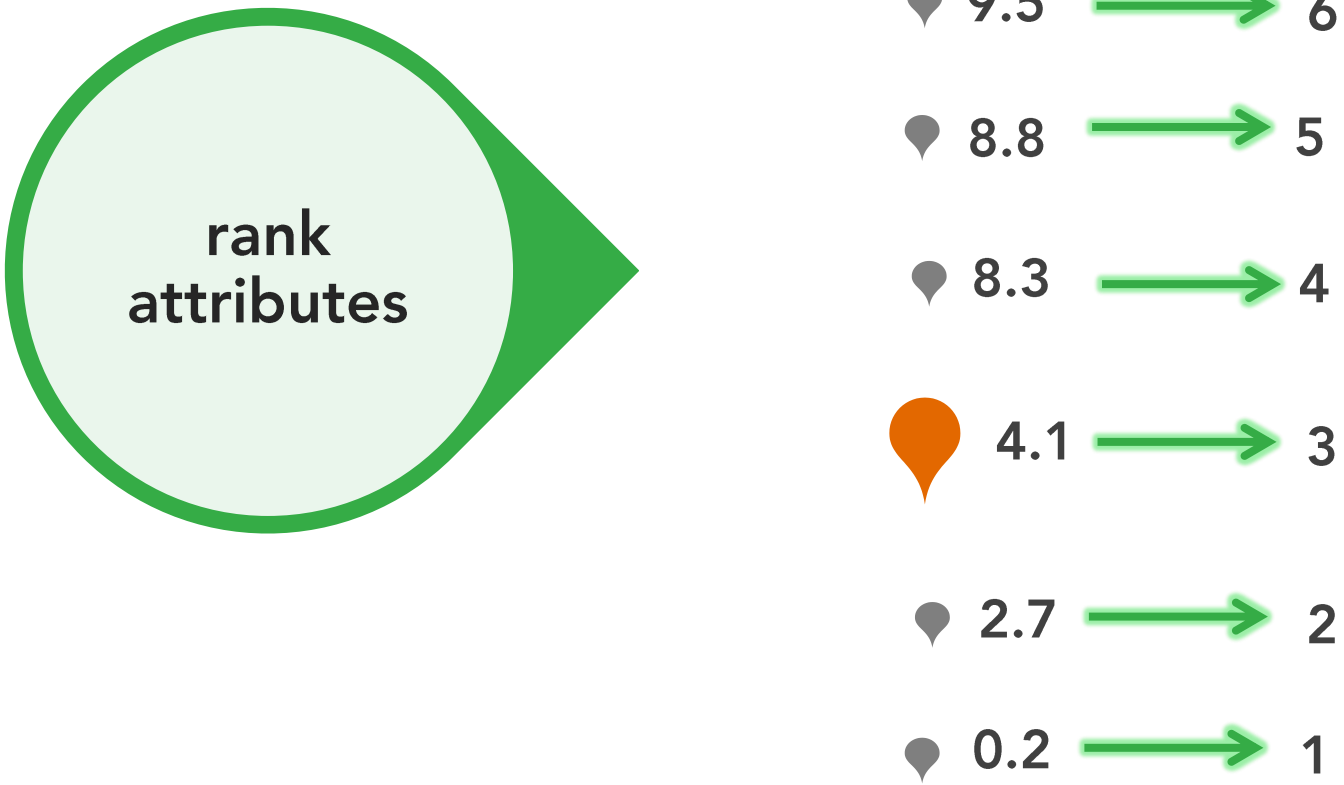


Ranked Attribute Values

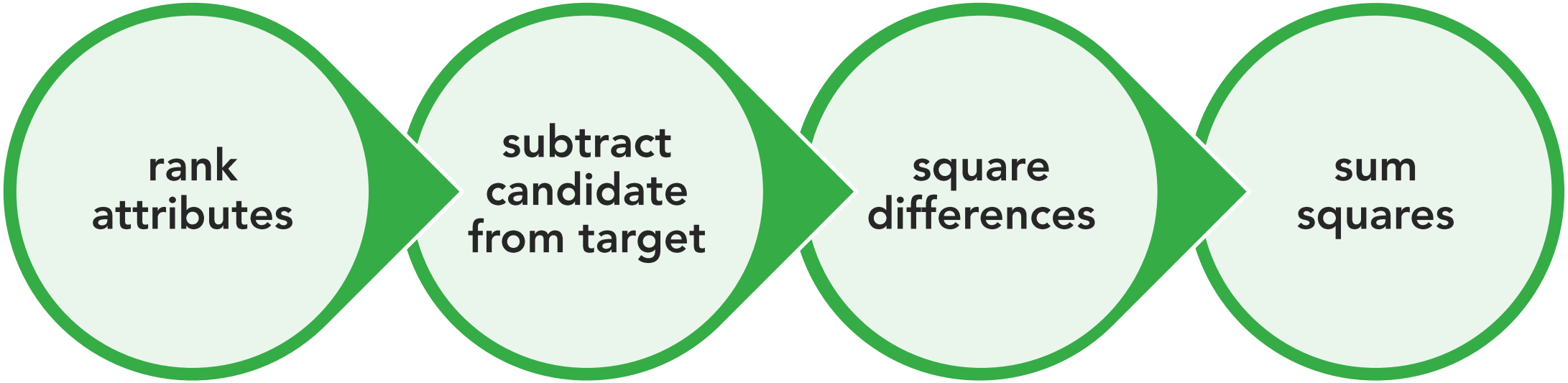
Ranked Attribute Values

Ranked Attribute Values

Ranked Attribute Values

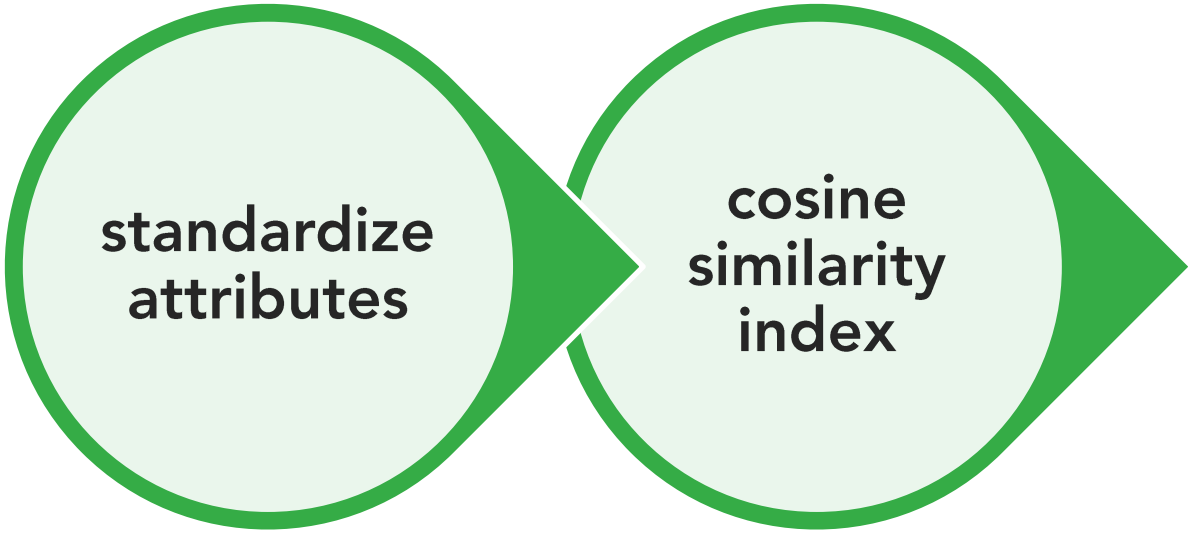


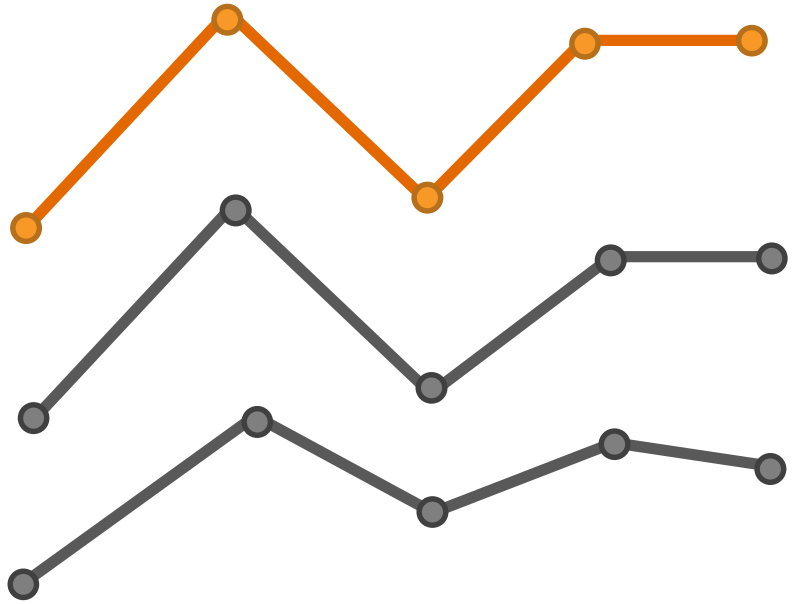
Ranked Attribute Values



Attribute Profiles

Attribute Profiles

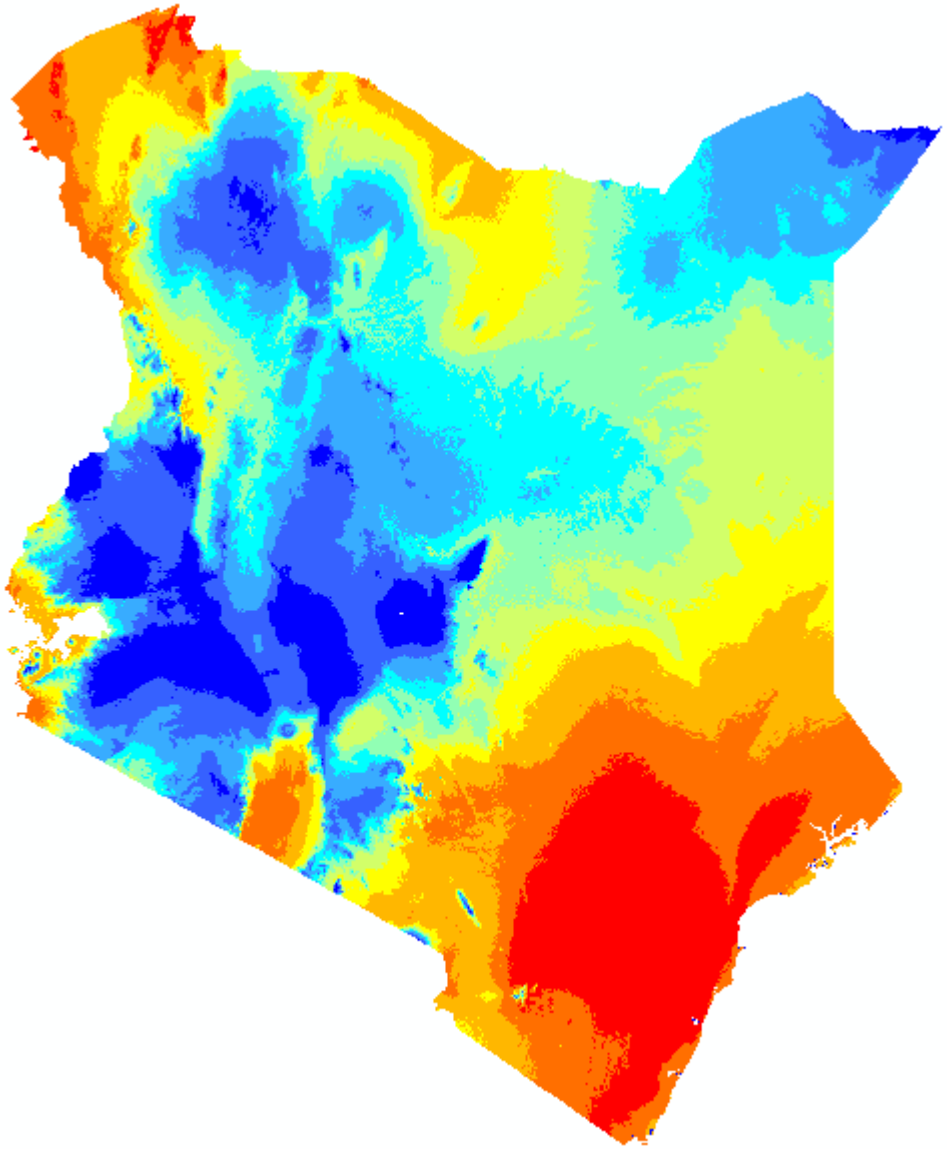




$$\text{cosine similarity index} = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} (A_i)^2} \sqrt{\sum_{i=1}^{n} (B_i)^2}}$$

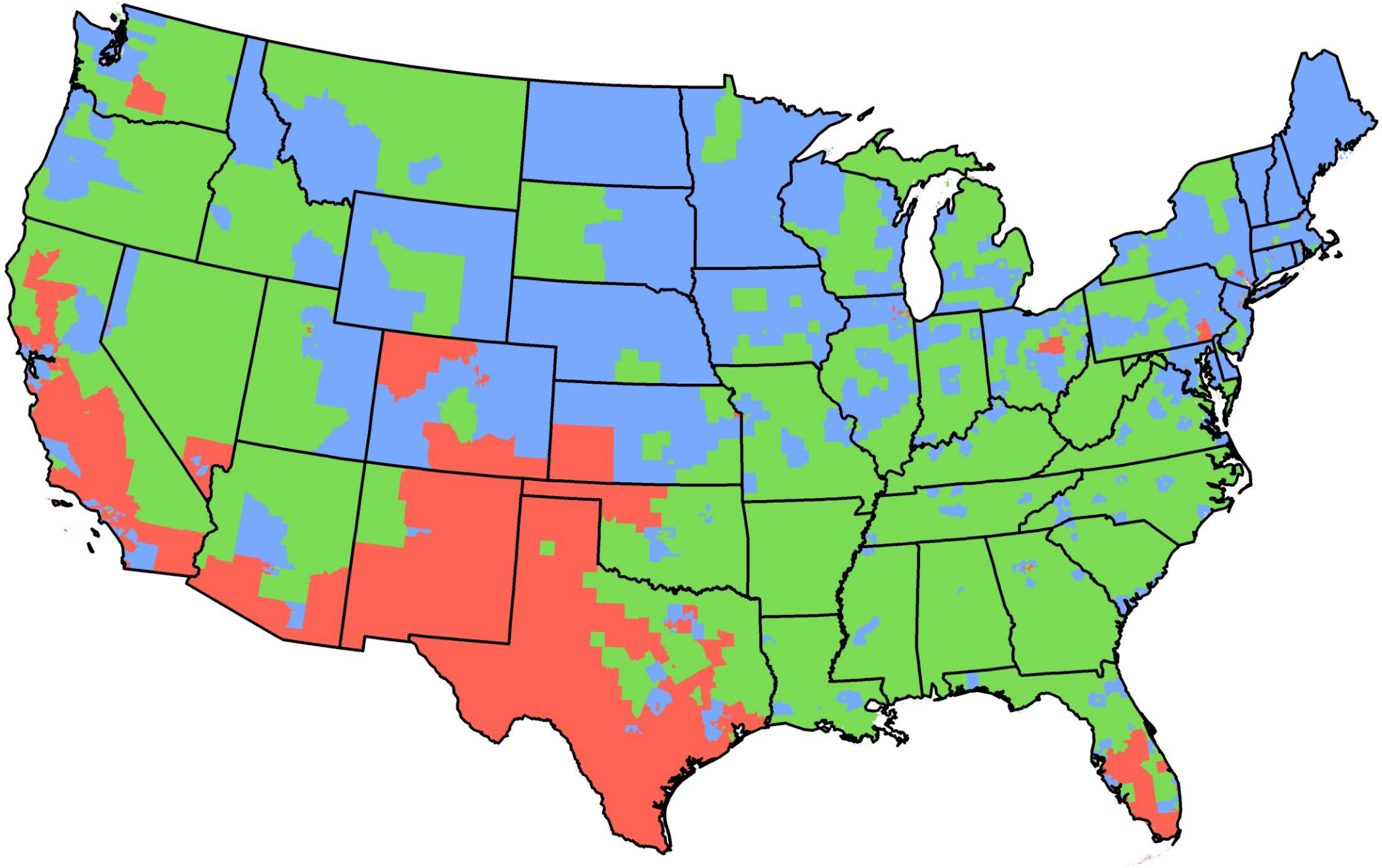
* Must have at least 2 attributes of interest

Dengue Fever Risk in Kenya

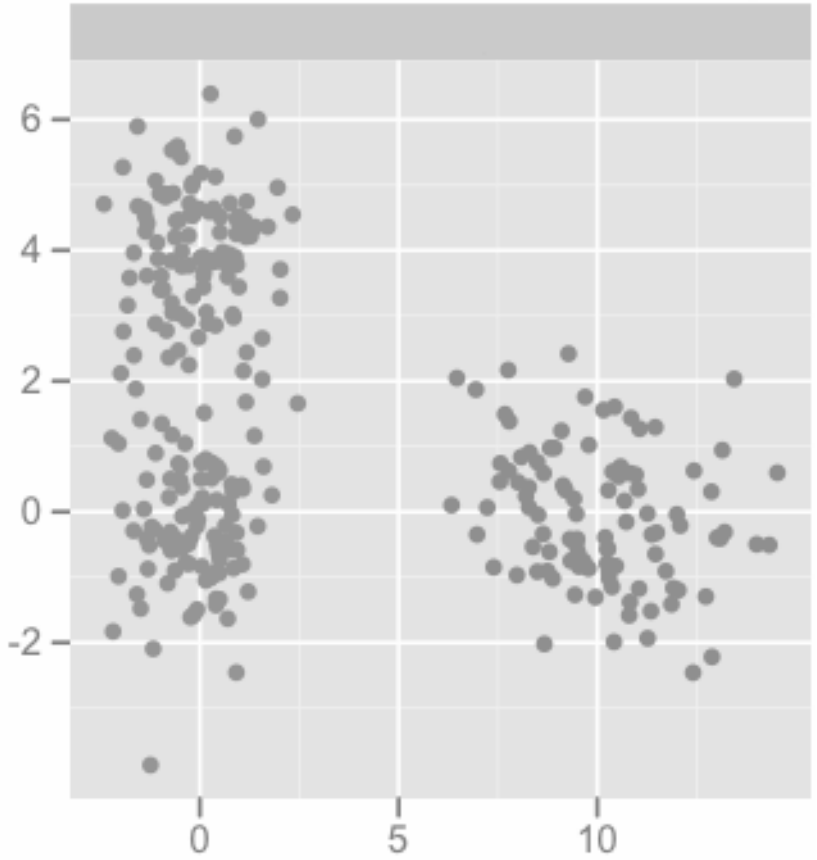


Grouping Analysis

groups features based on feature attributes and optional spatial/temporal constraints

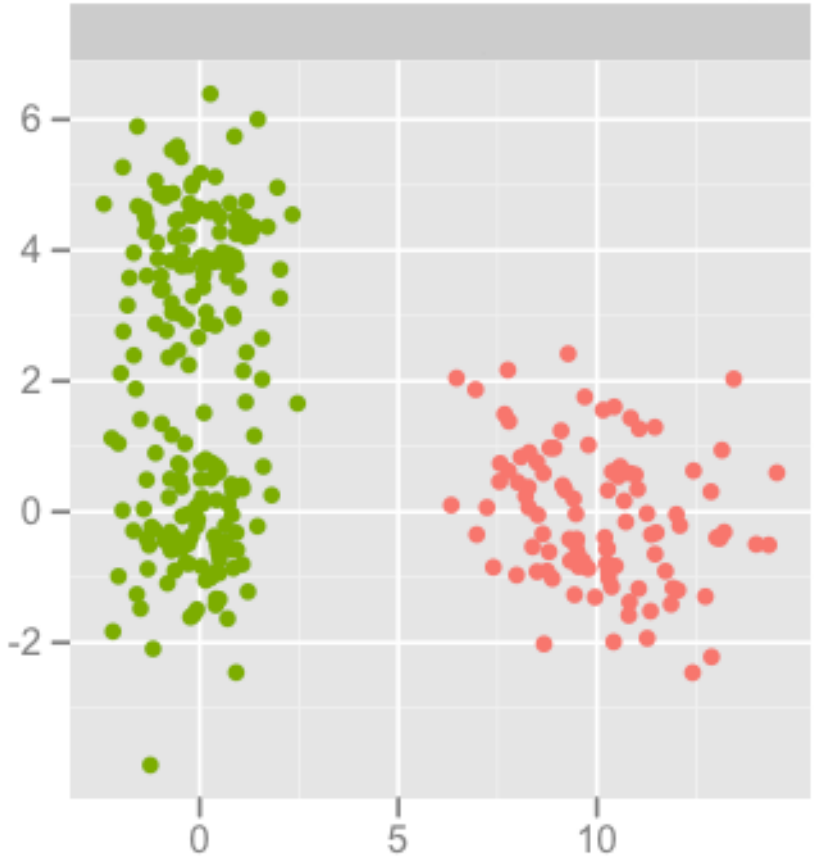


K Means

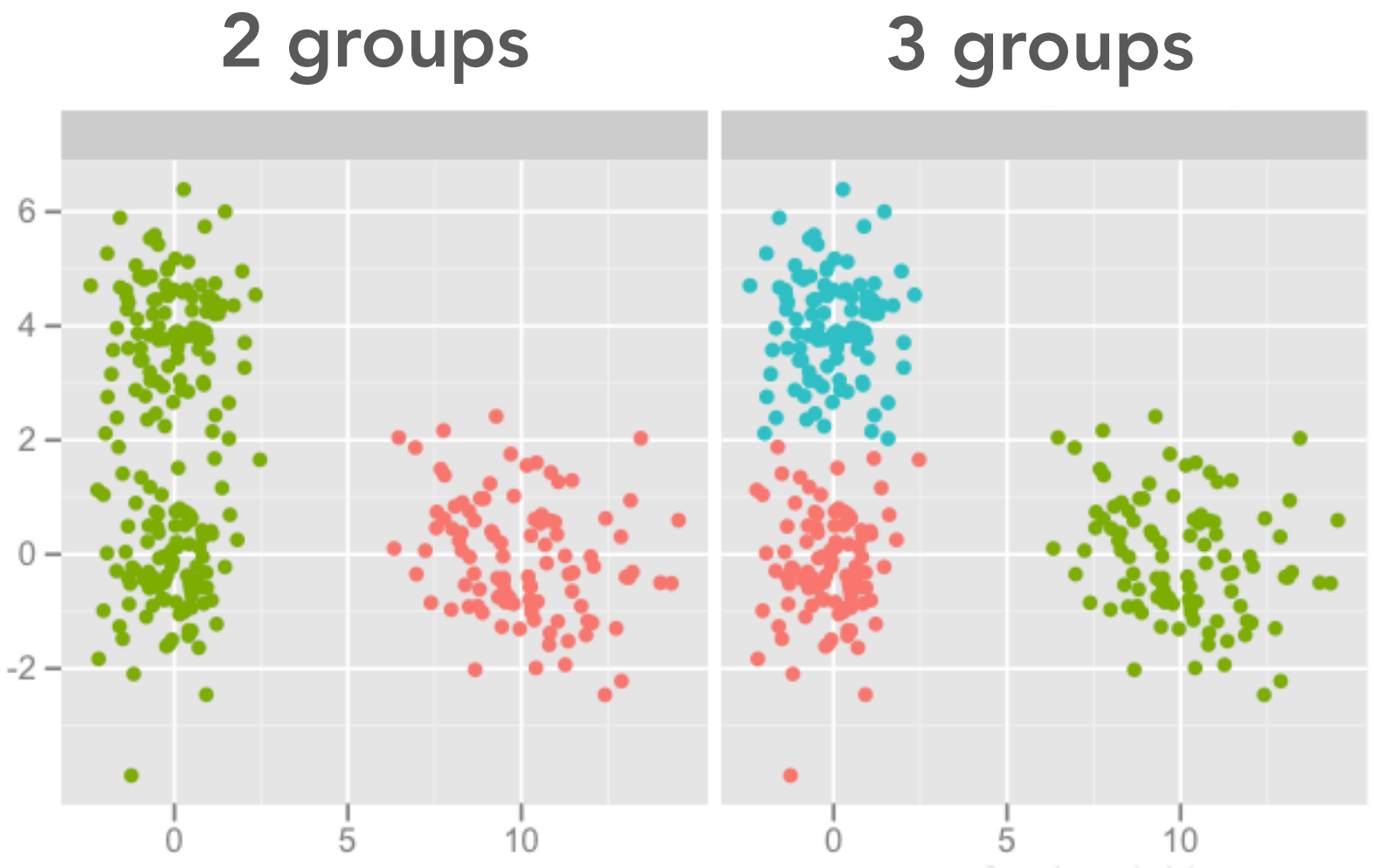


K Means

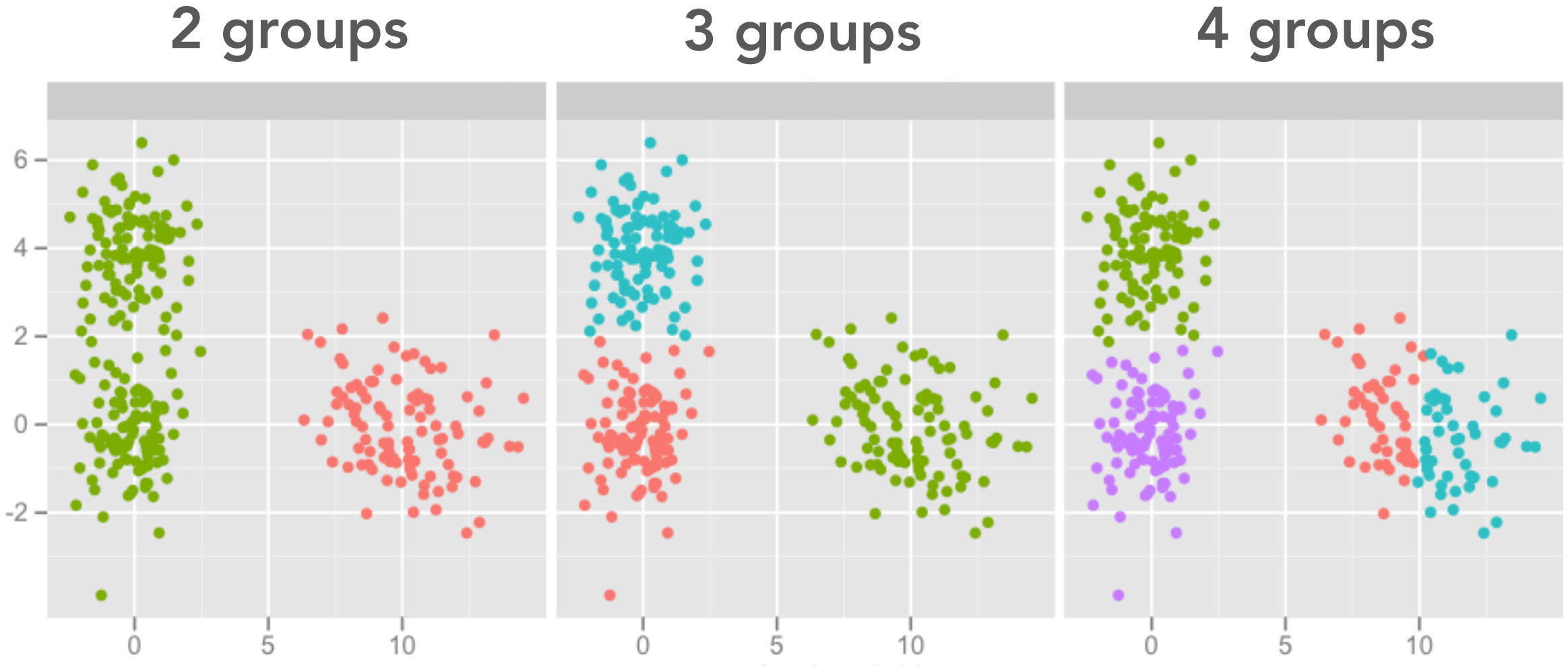
2 groups



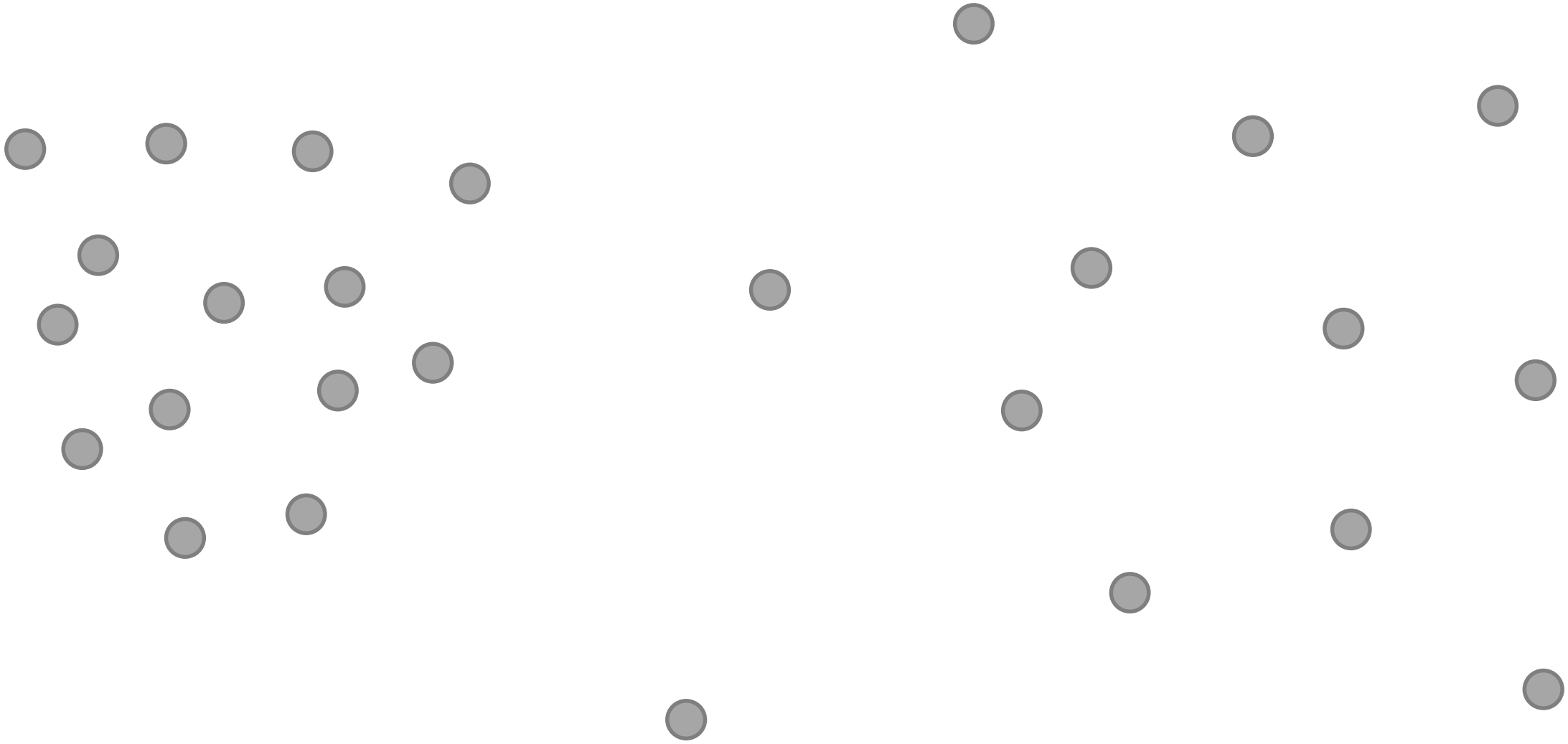
K Means



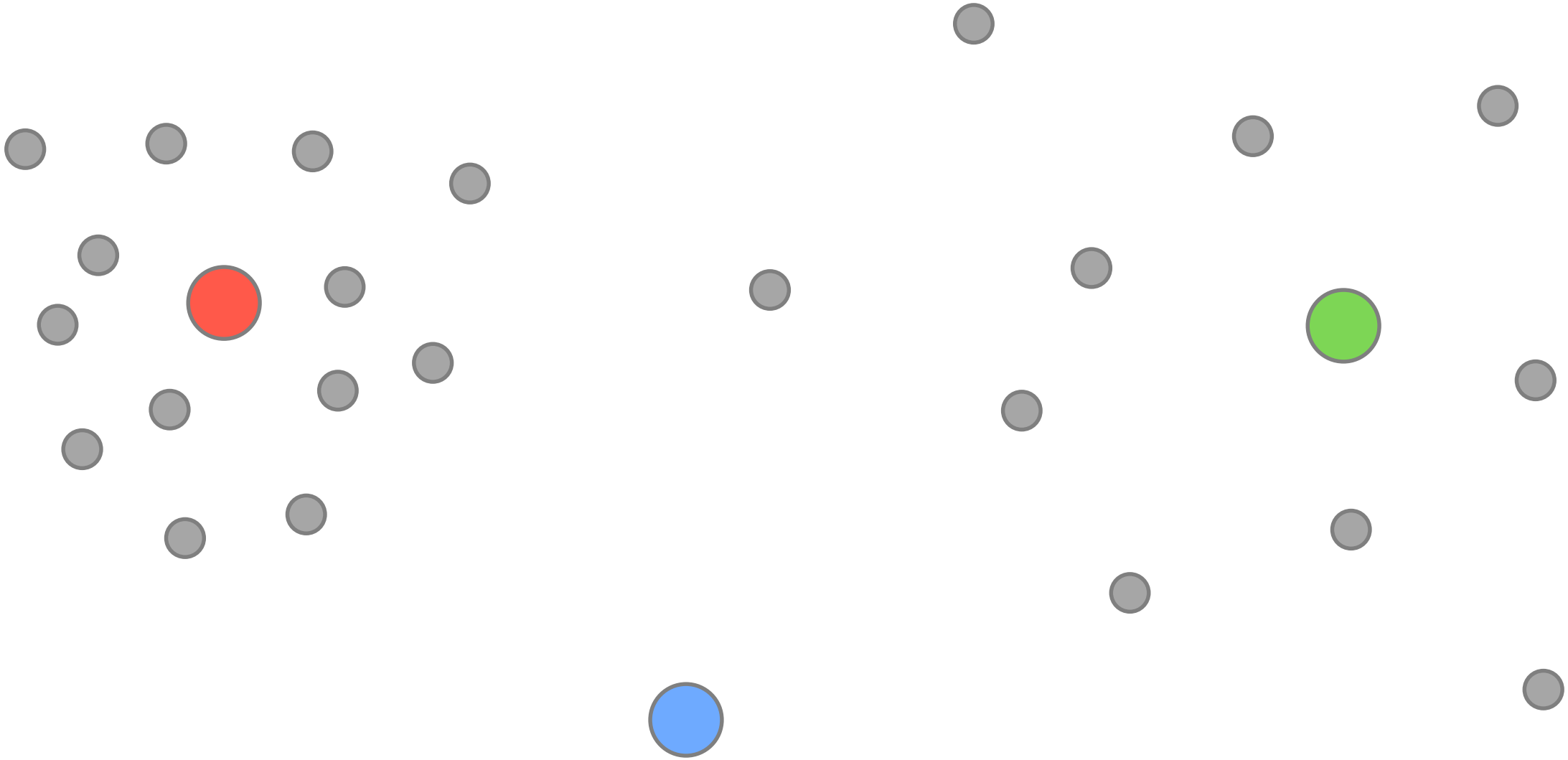
K Means



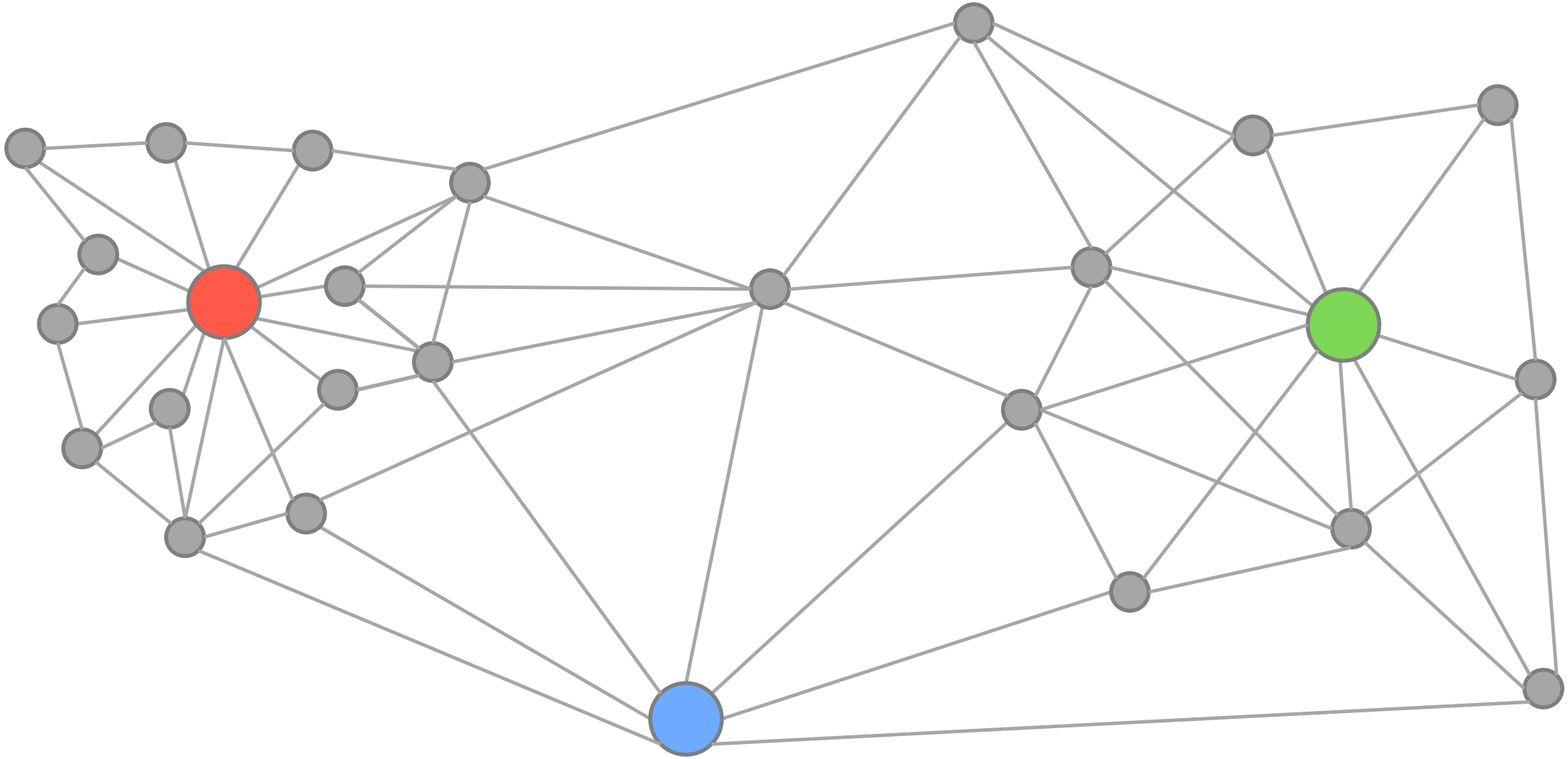
Minimum Spanning Tree



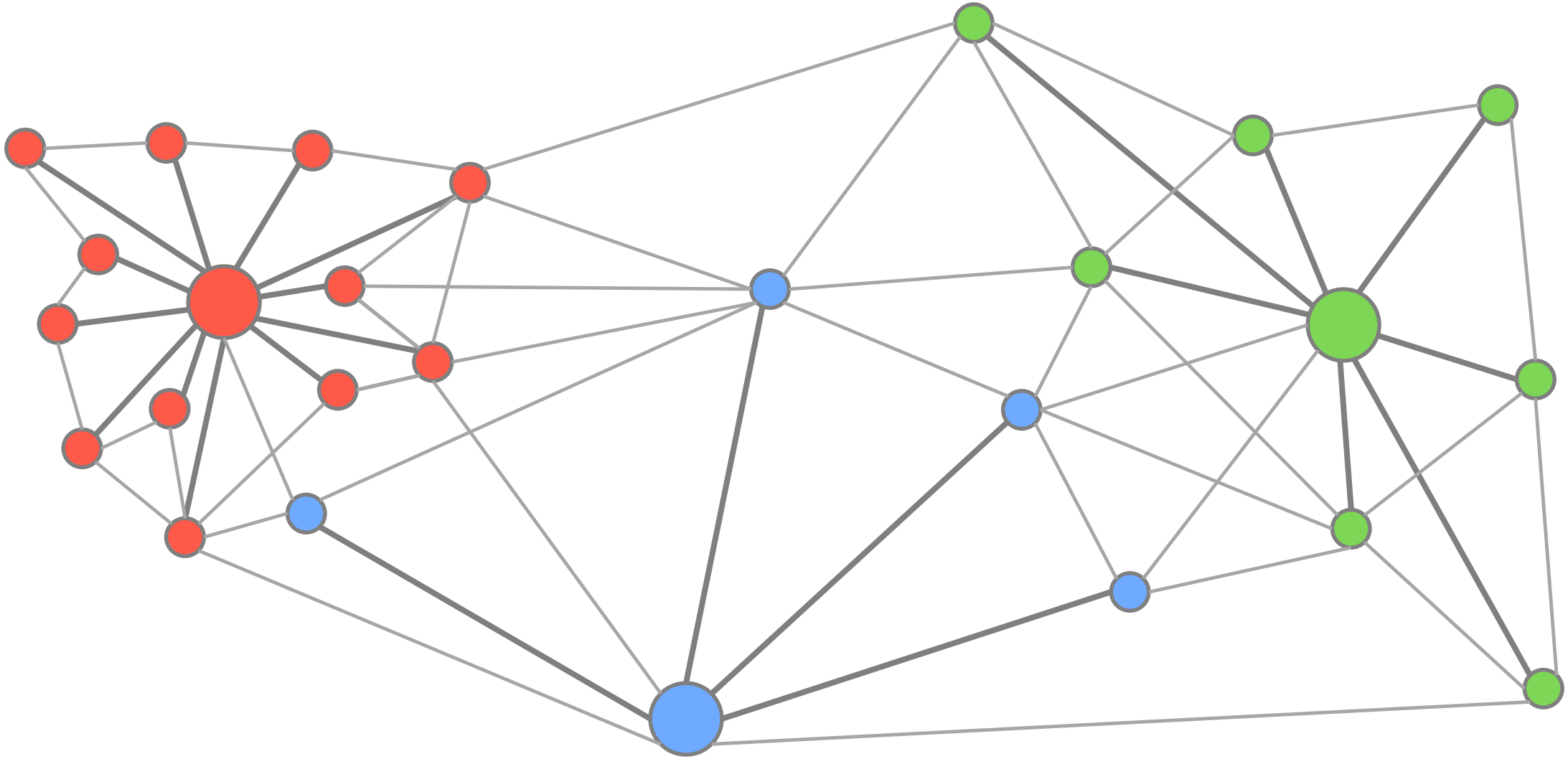
Minimum Spanning Tree



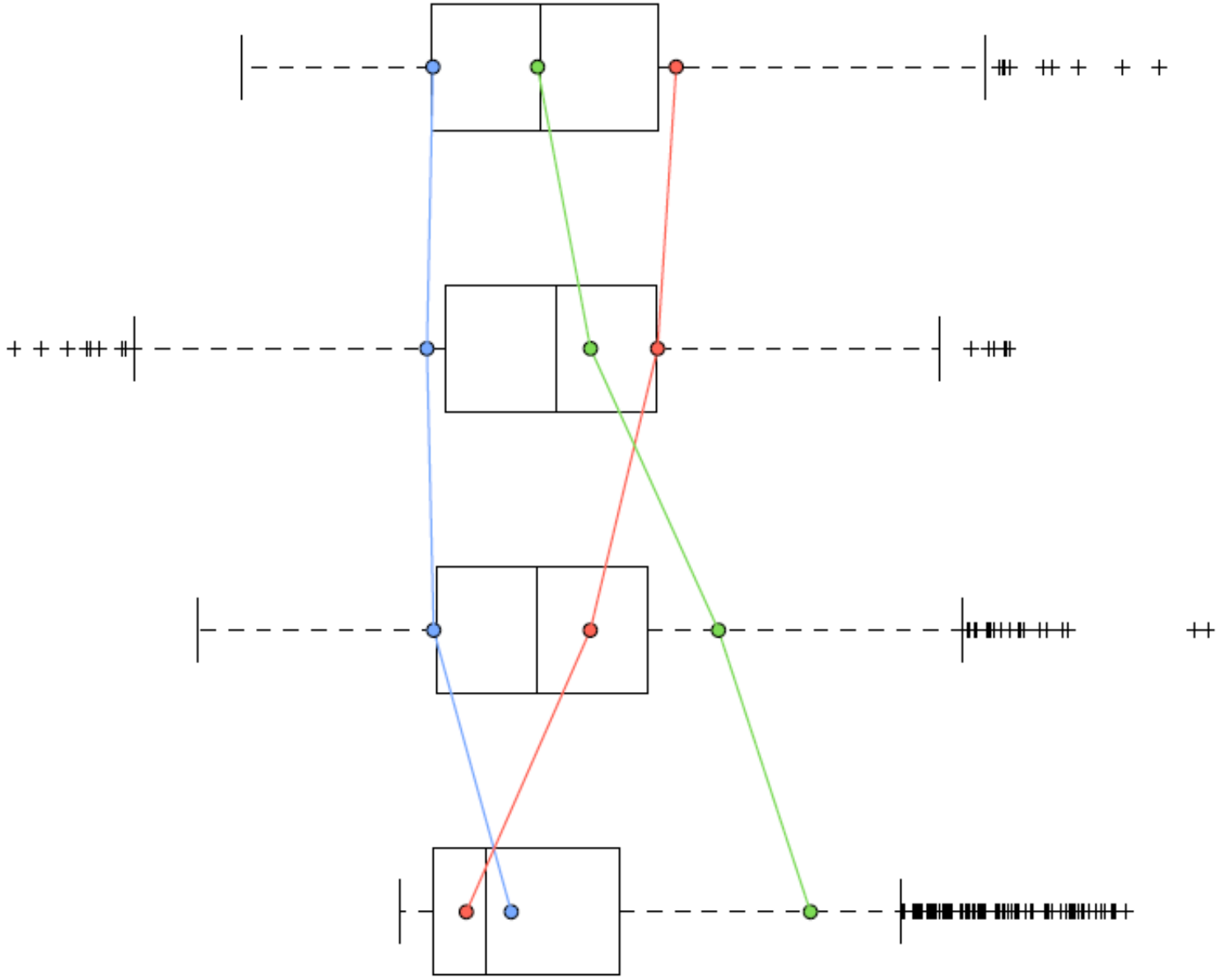
Minimum Spanning Tree



Minimum Spanning Tree



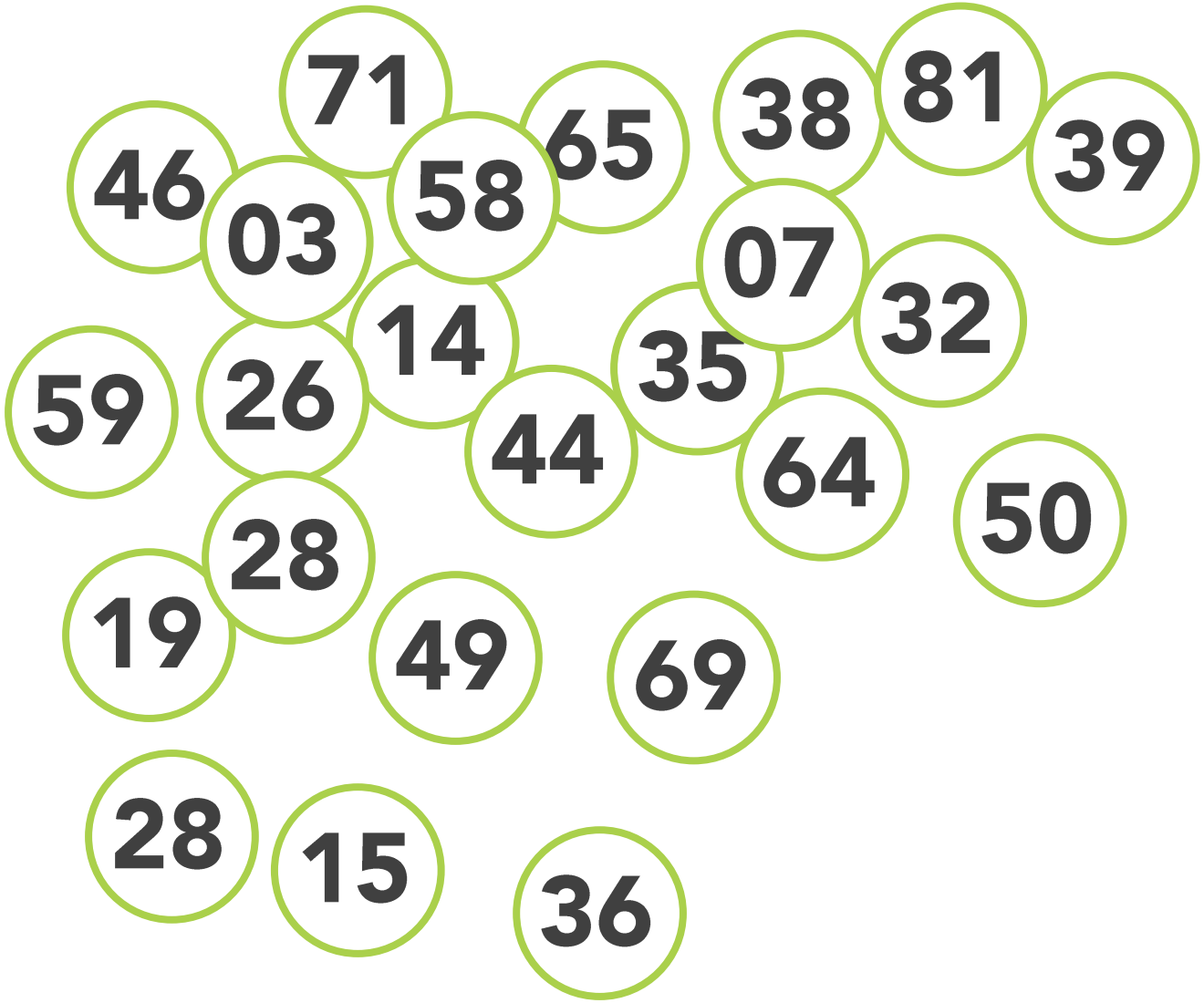
interpret results through box plots



Hot Spot Analysis
(Getis-Ord Gi*)

given a set of weighted features, identifies statistically significant hot spots and cold spots using the Getis-Ord Gi* statistic

14 15 92 65 35 89 79 32 38
46 26 43 38 32 79 50 28 84
19 71 69 39 93 75 10 58 20
97 49 44 59 23 07 81 64 06
28 62 08 99 86 28 03 48 25


71
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81
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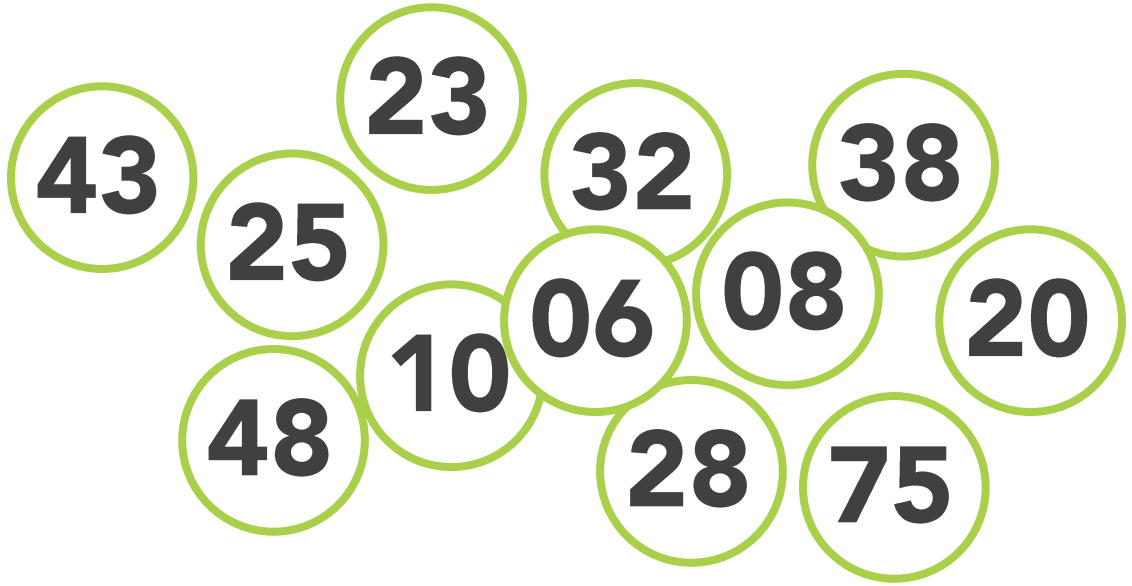
28
15
36


23
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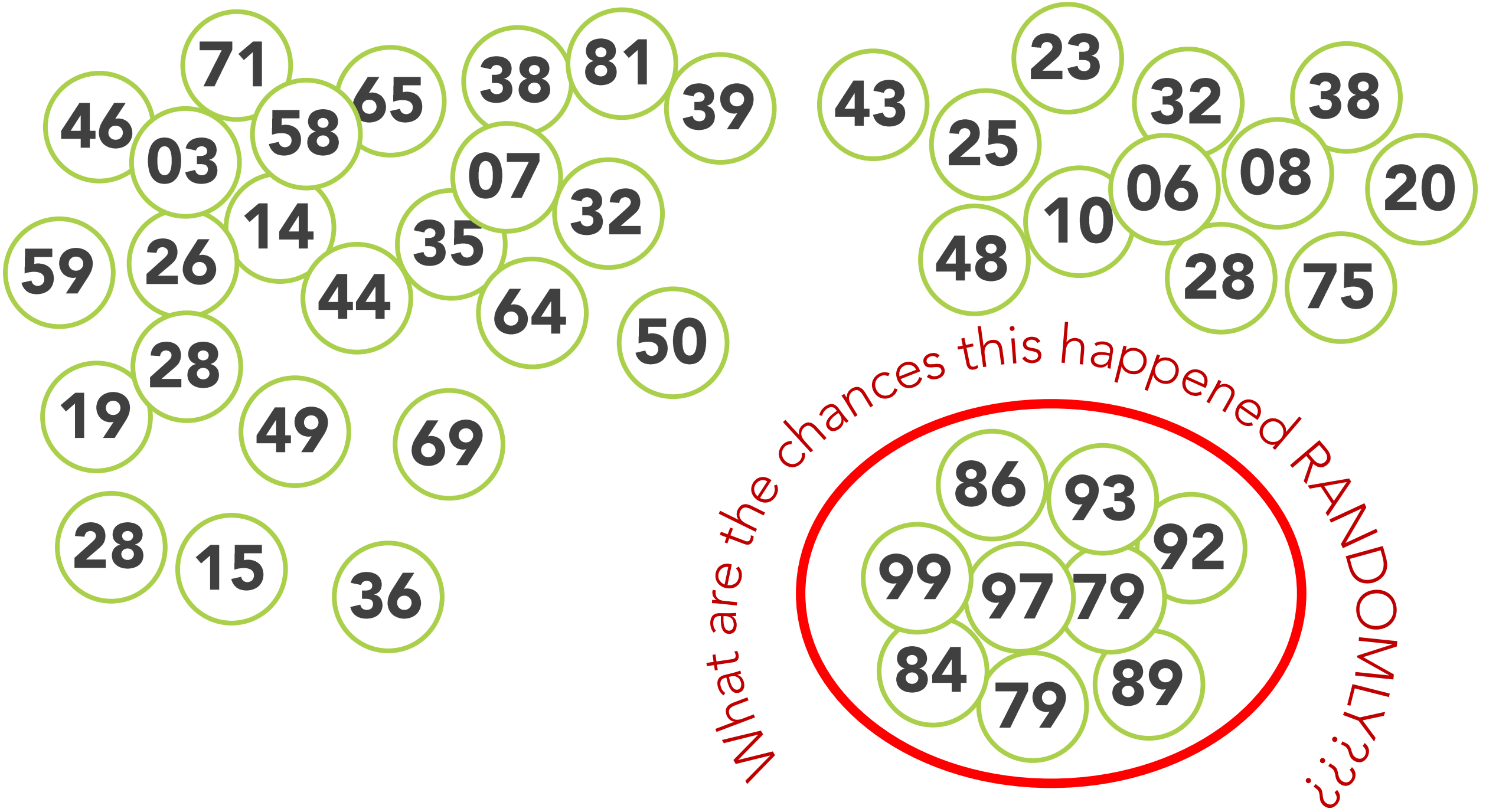
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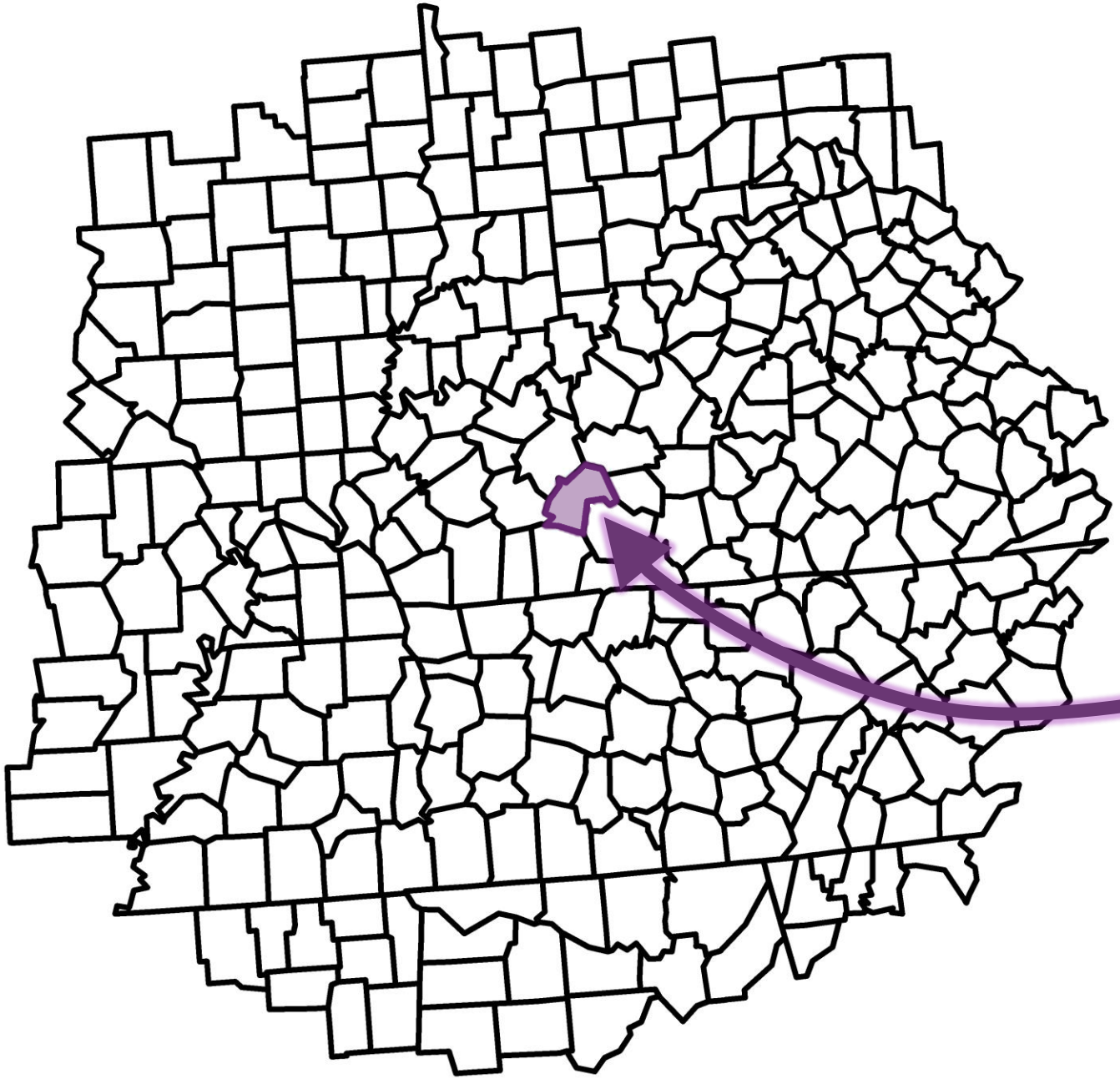
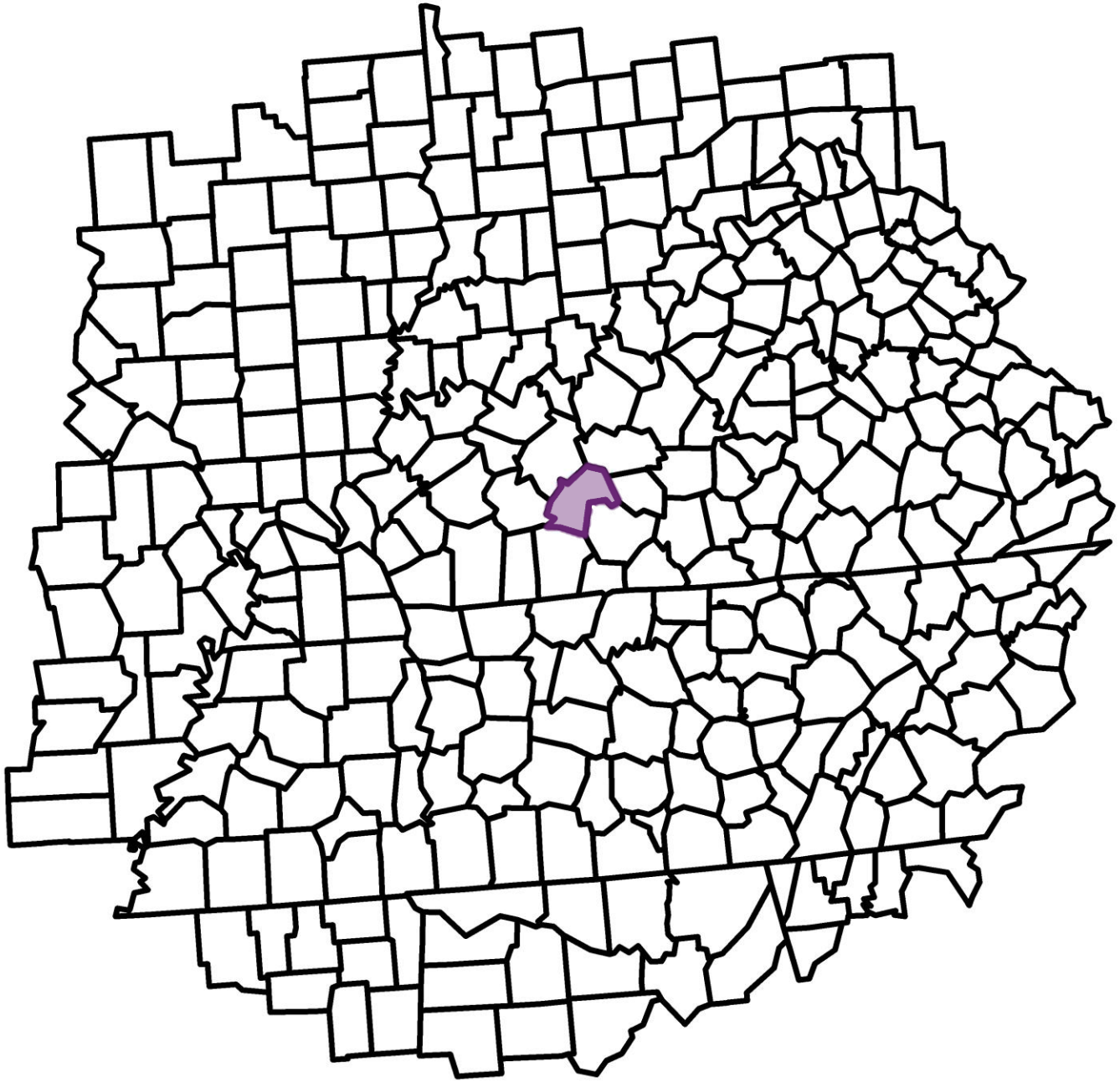
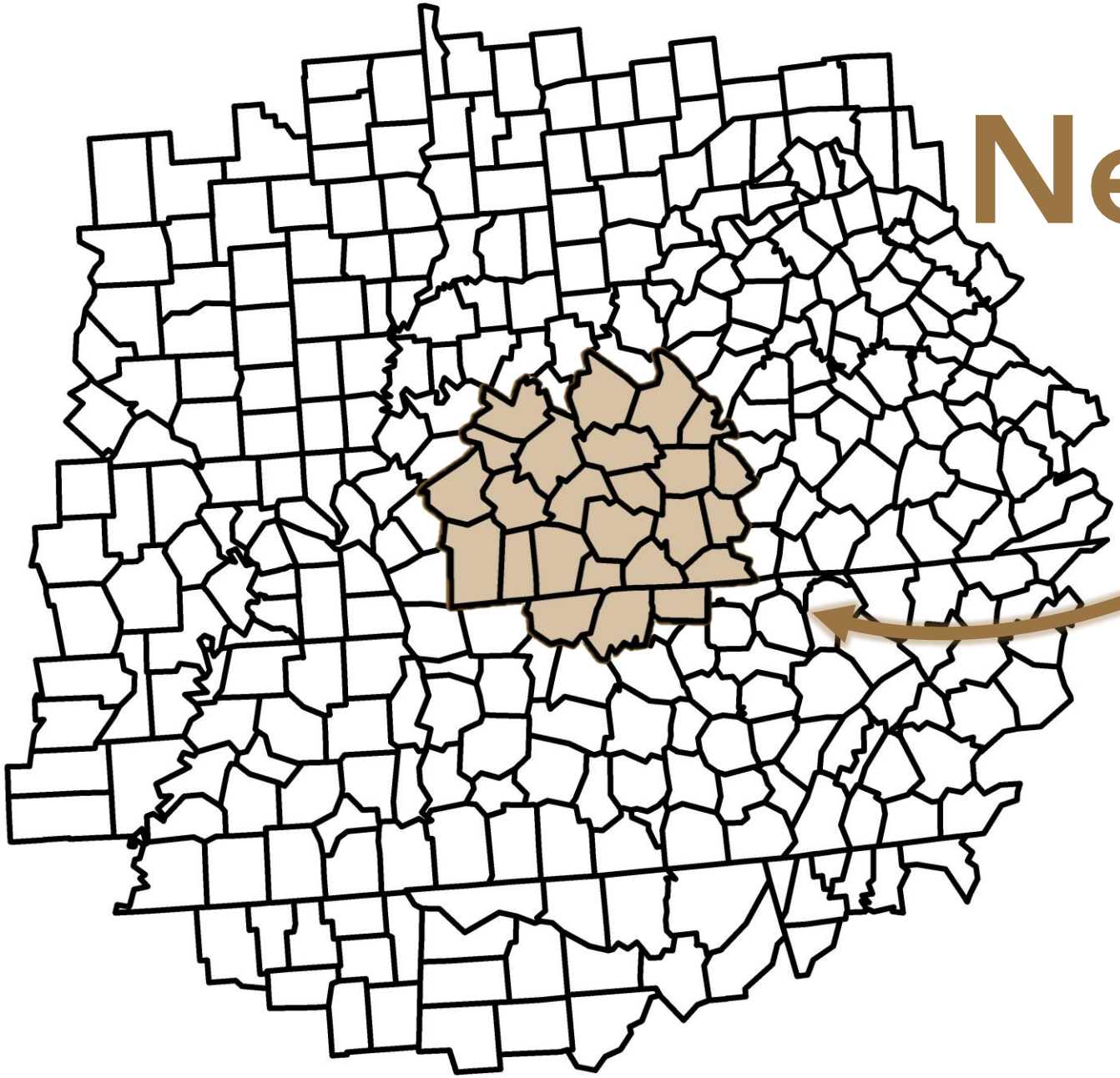
What are the chances this happened RANDOMLY???
86
93
92
99
97
79
84
79
89



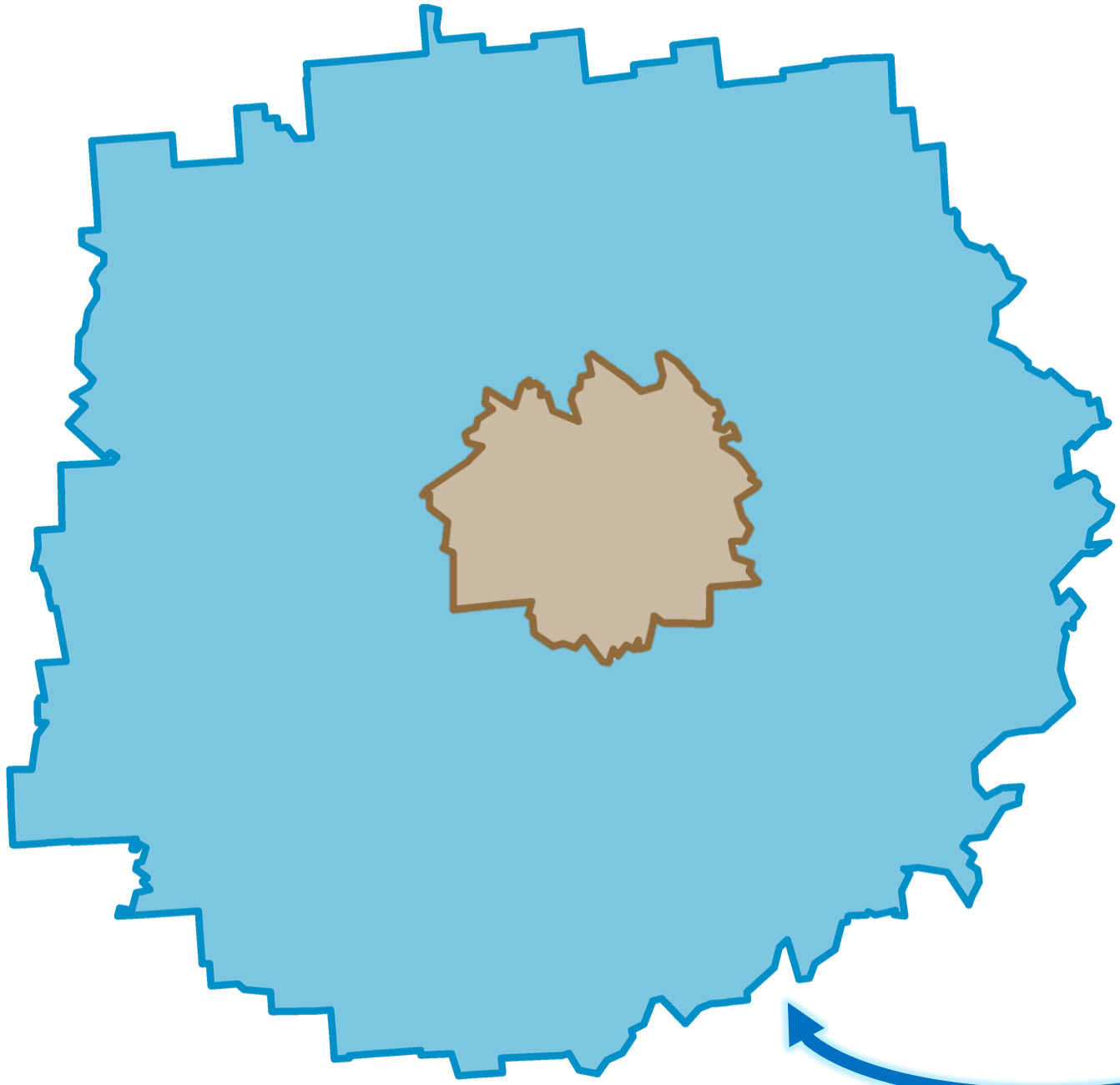
feature



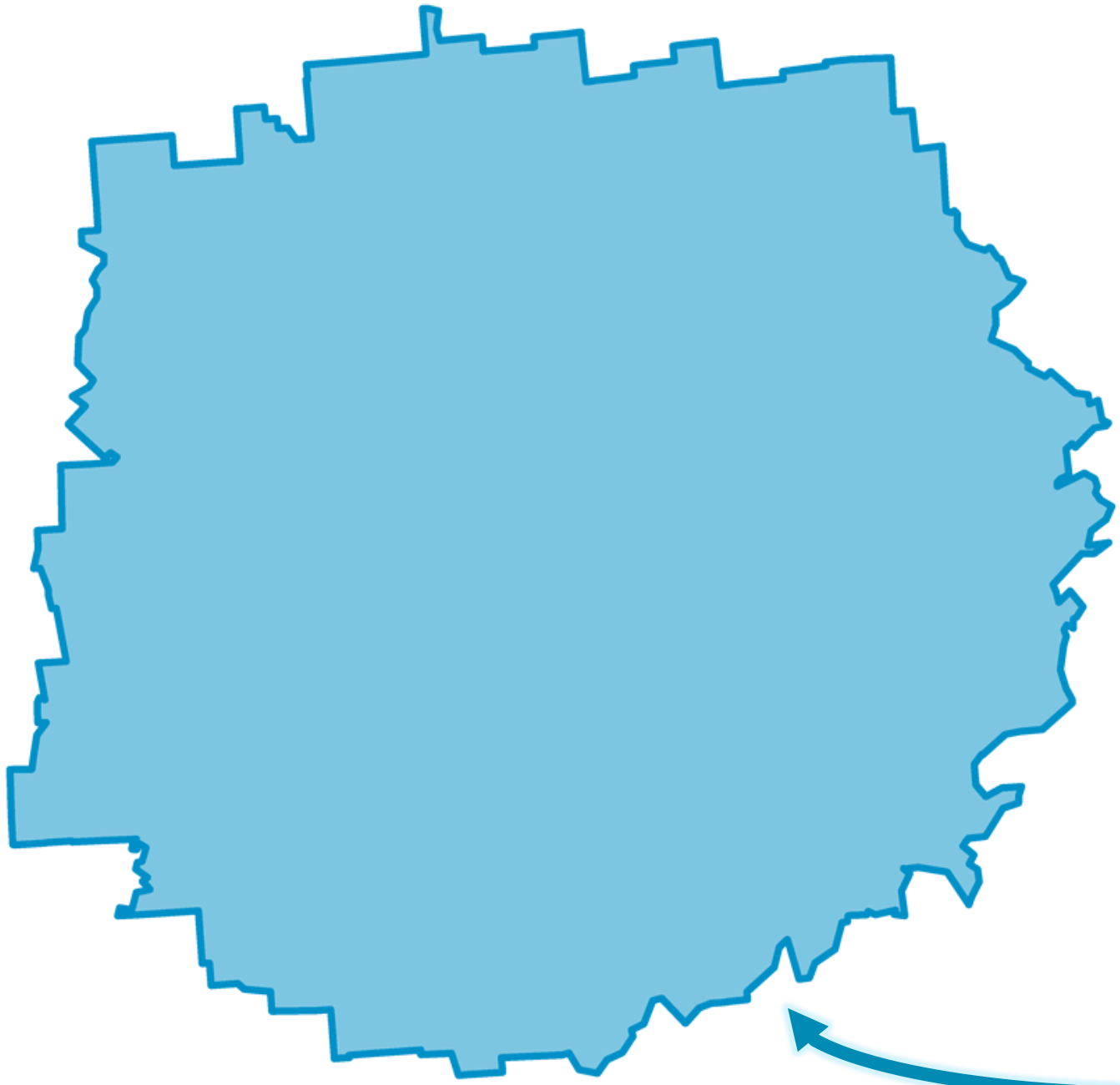
each feature has a **value**



Neighborhood

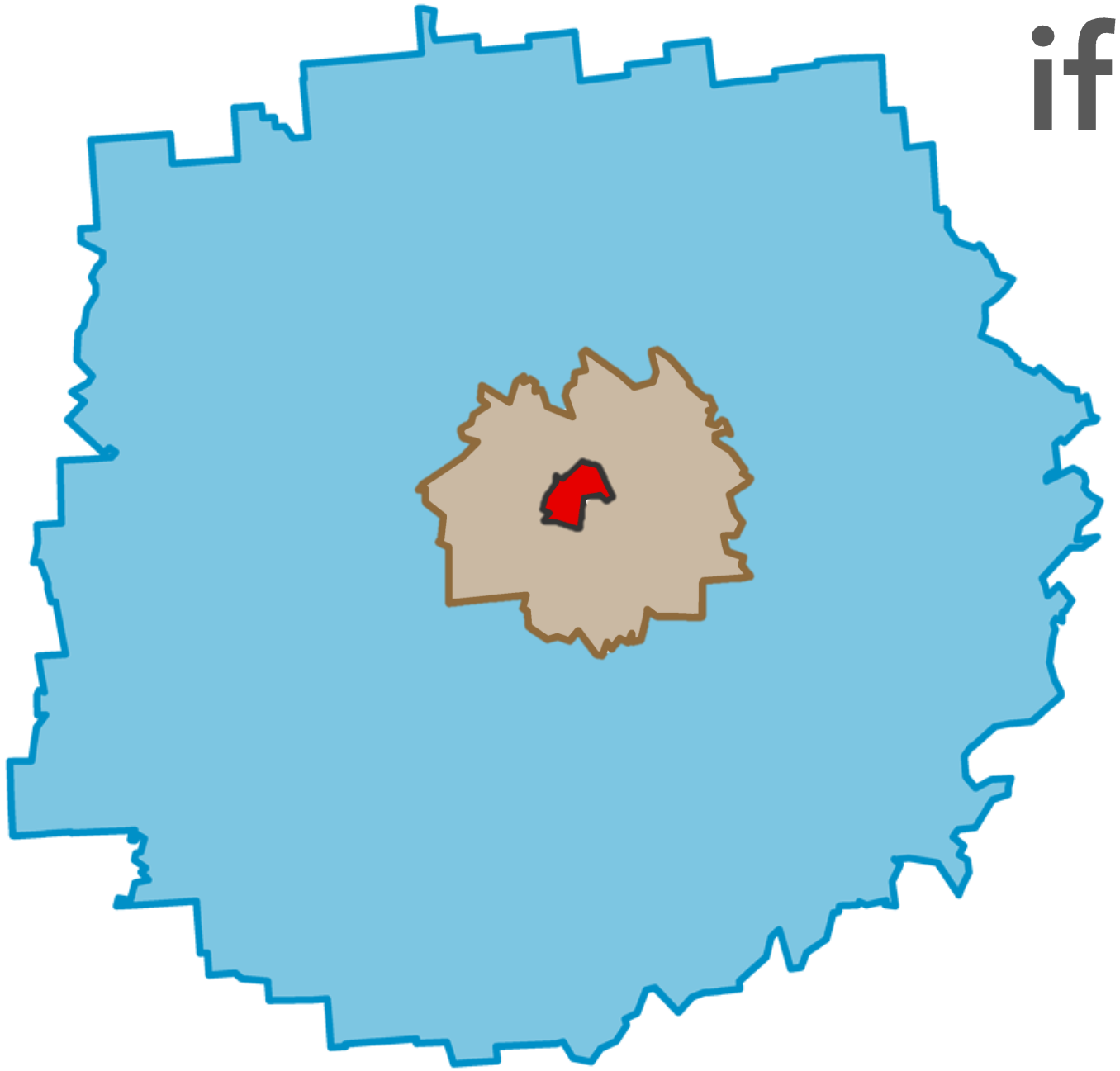


Study Area



is this

significantly different from this?



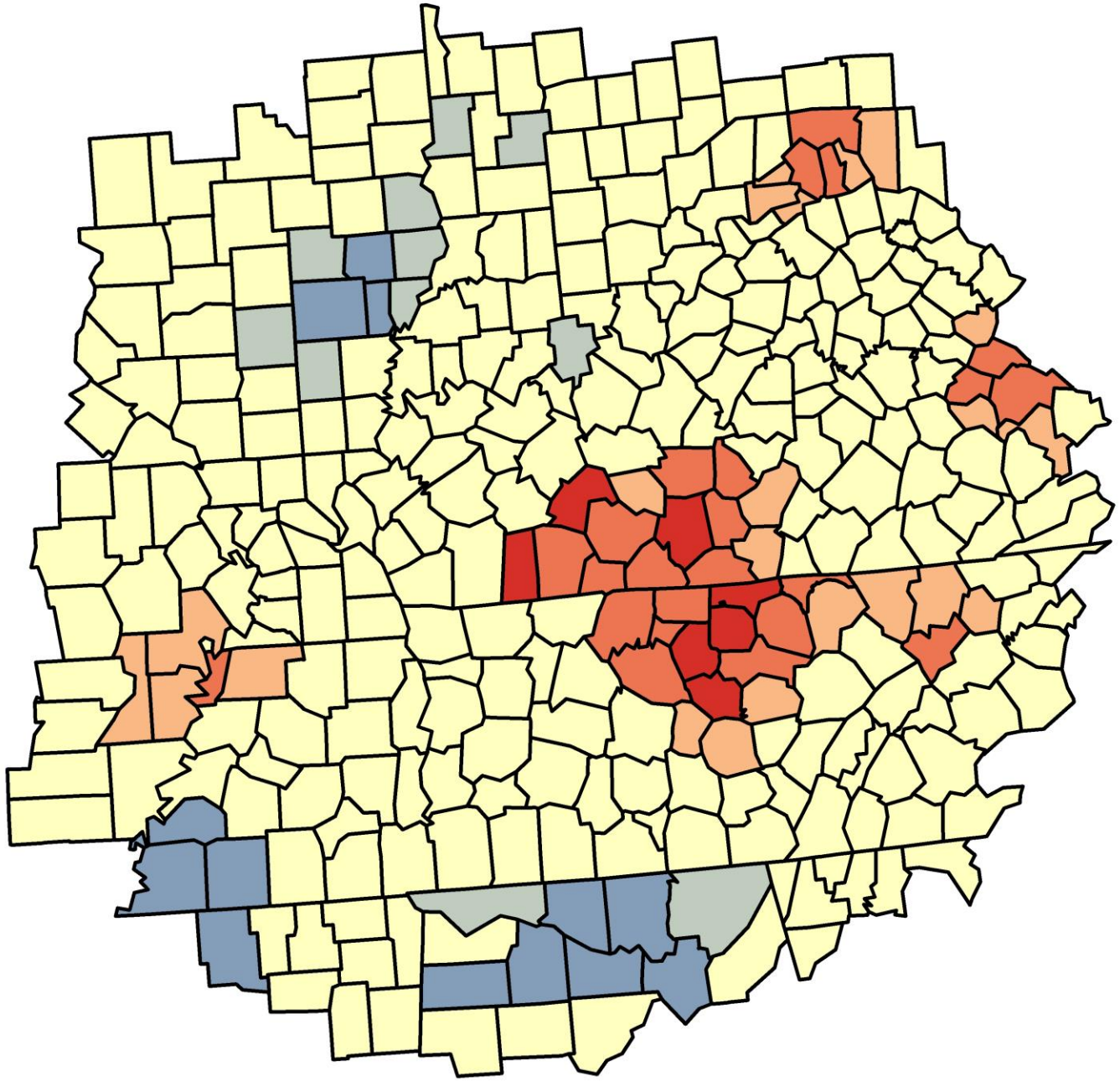
if significantly higher…

feature is marked as a **hot spot**!

Hot Spot - 90% Confidence

Hot Spot - 95% Confidence

Hot Spot - 99% Confidence



Cold Spot - 99% Confidence

Cold Spot - 95% Confidence

Cold Spot - 90% Confidence

Not Significant

Hot Spot - 90% Confidence

Hot Spot - 95% Confidence

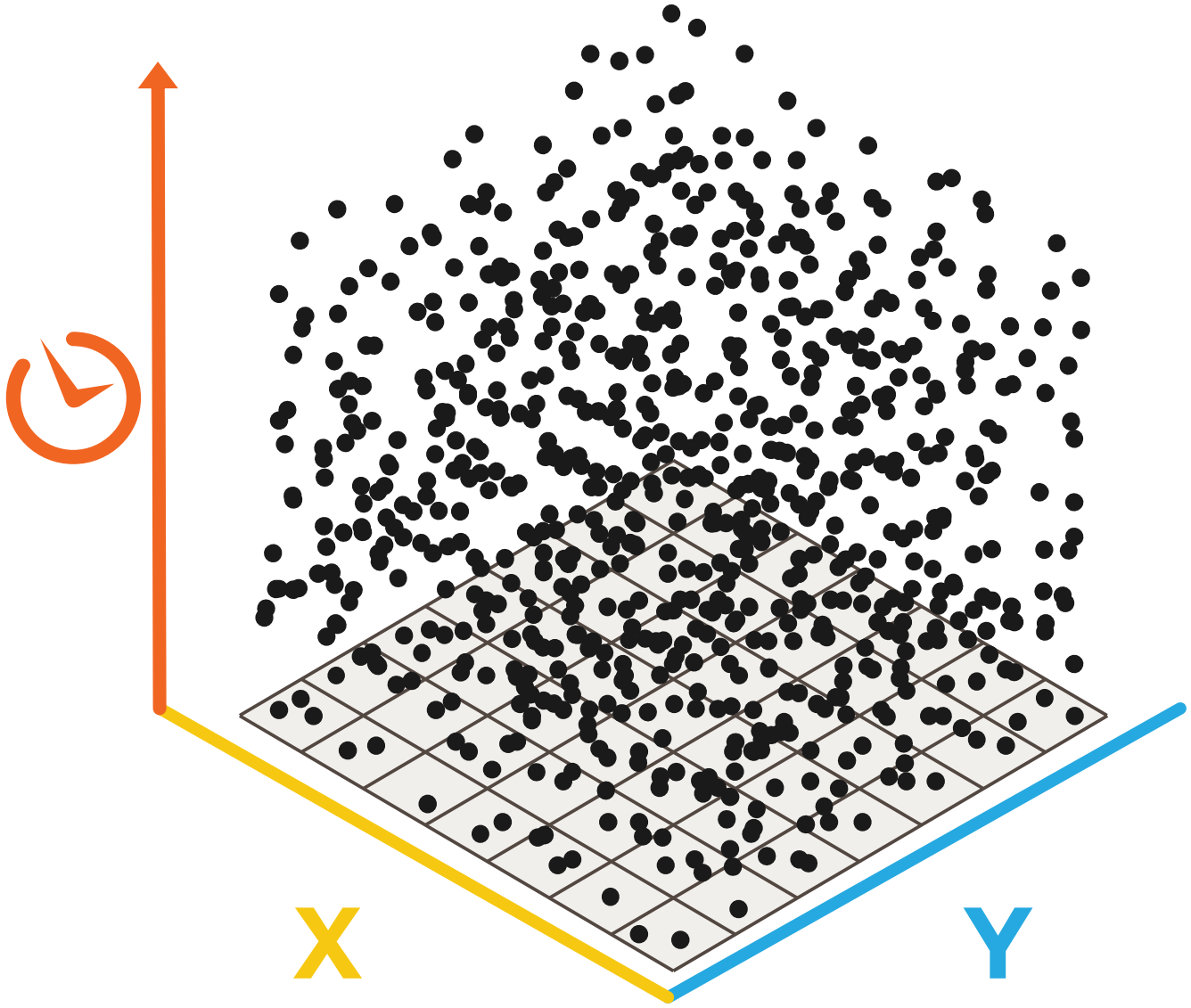
Hot Spot - 99% Confidence

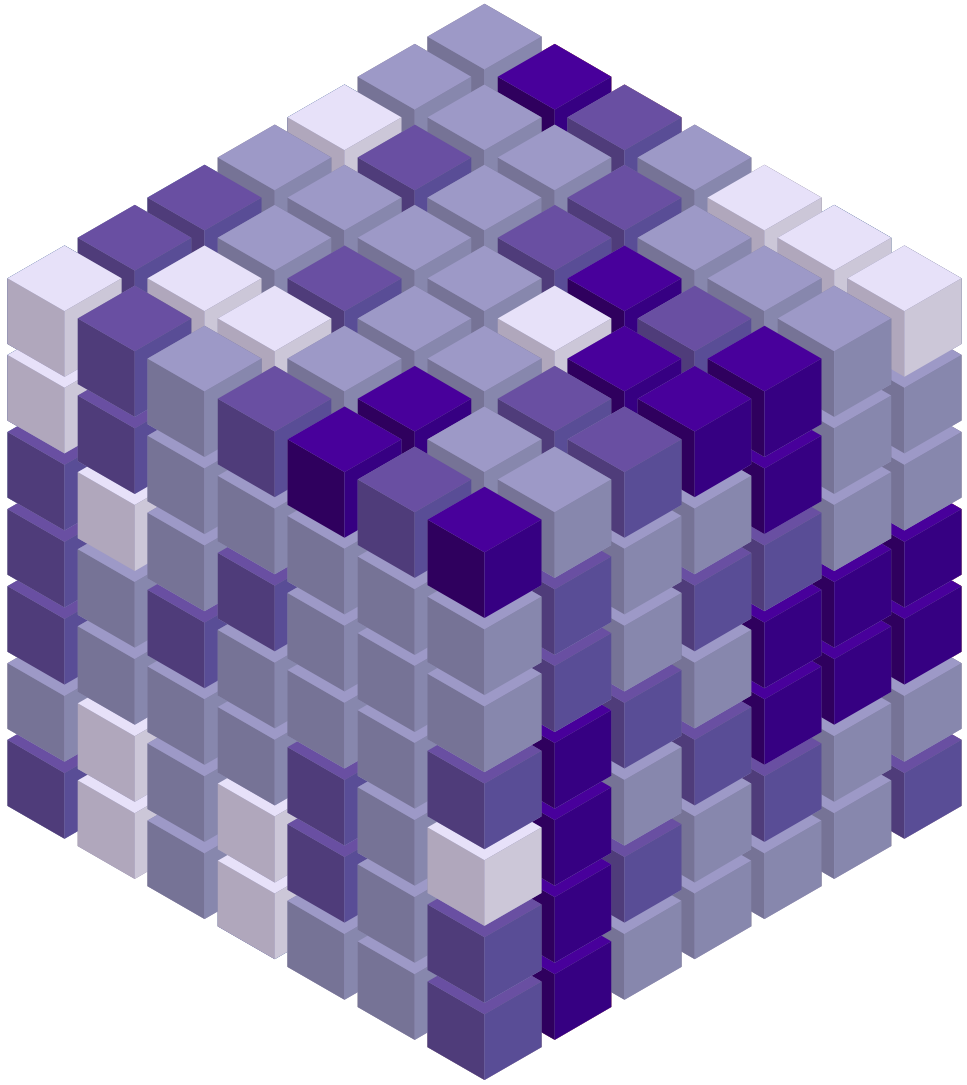
Optimized Hot Spot Analysis

evaluates input feature characteristics to produce optimal Hot Spot Analysis results

Create Space Time Cube

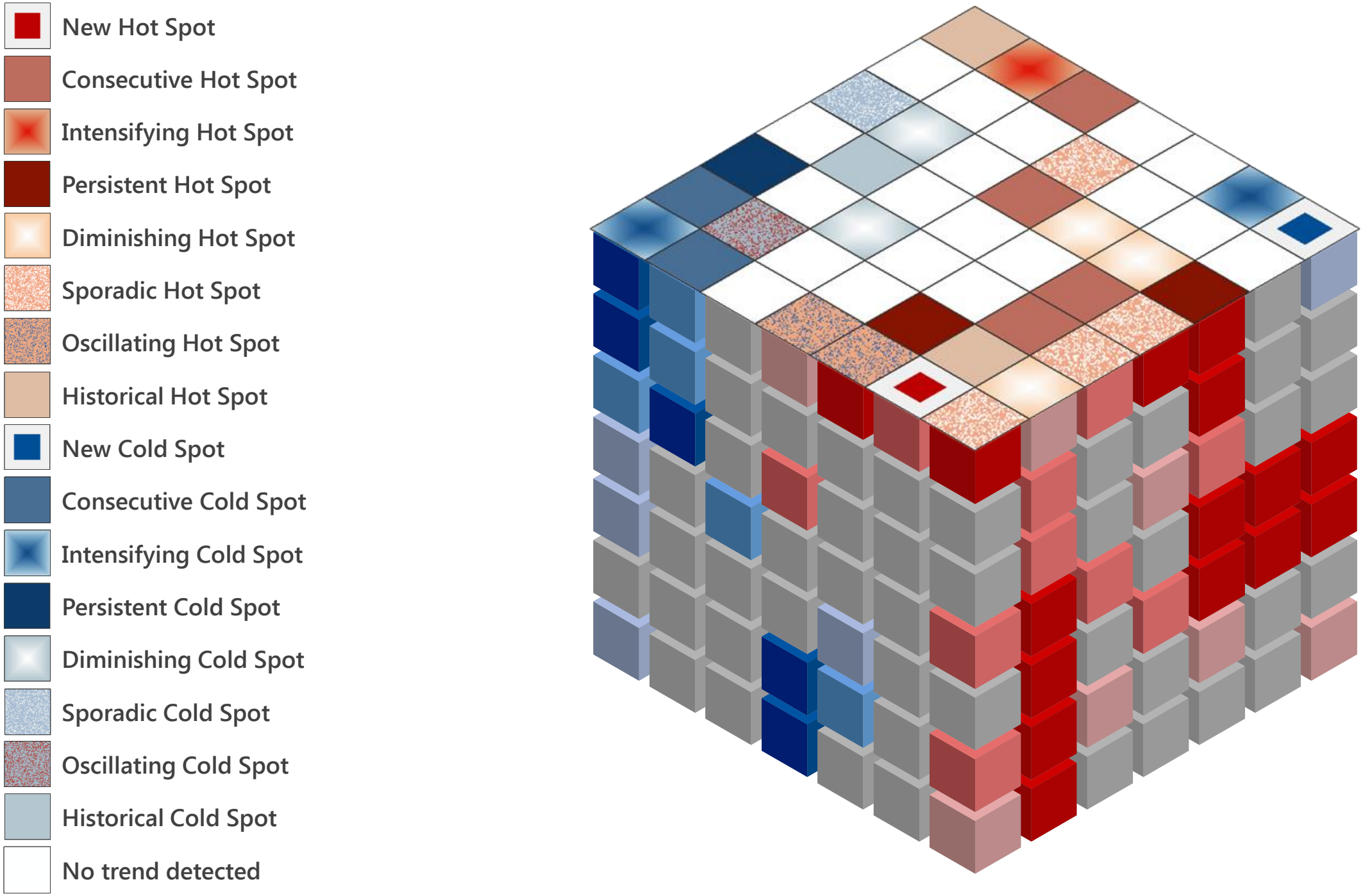
aggregates a set of points into space-time bins, counting the number of points and optionally calculating statistics for attributes


X
Y



Emerging Hot Spot Analysis

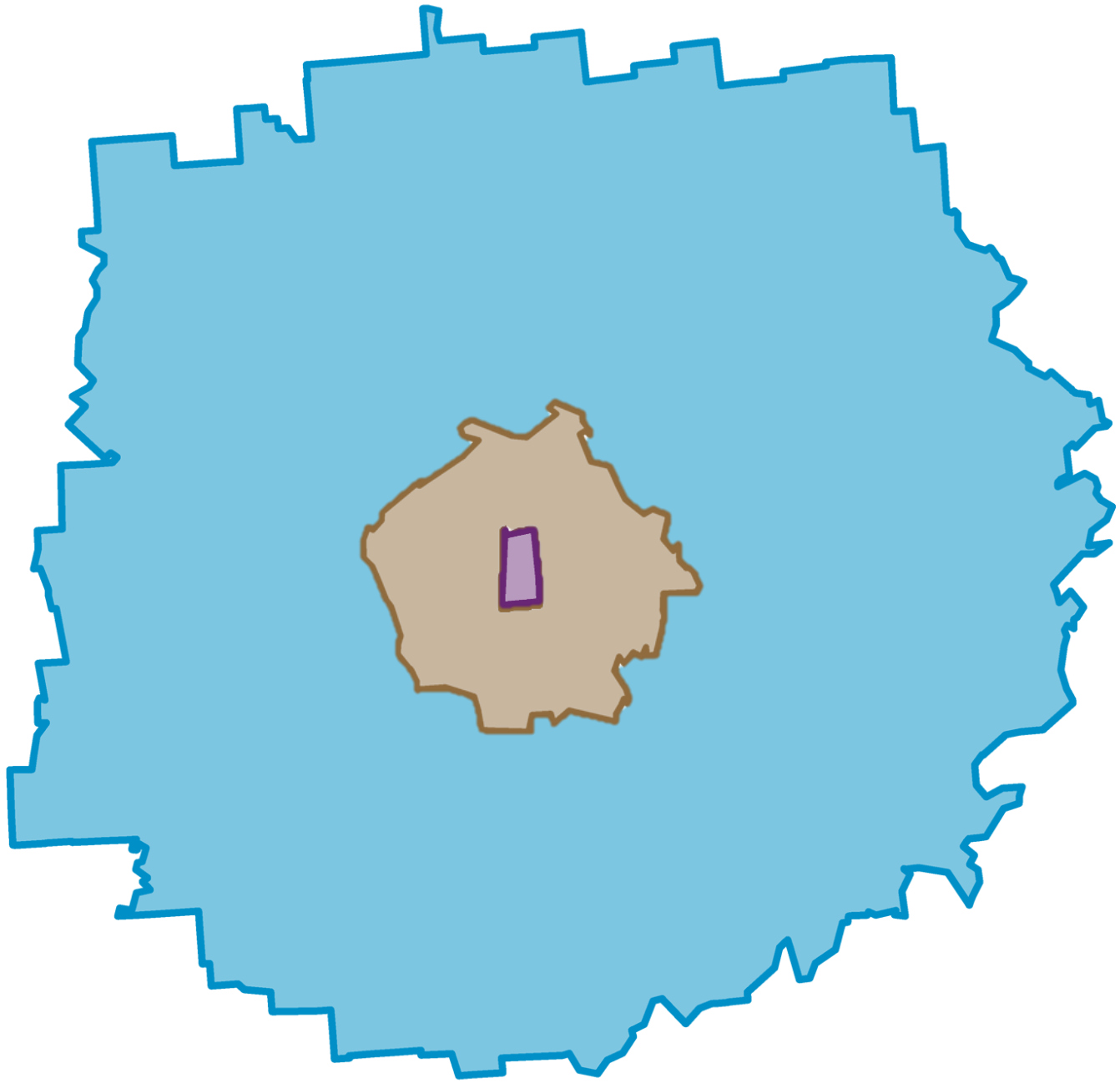
identifies trends in the clustering of counts or summary fields in a space time cube

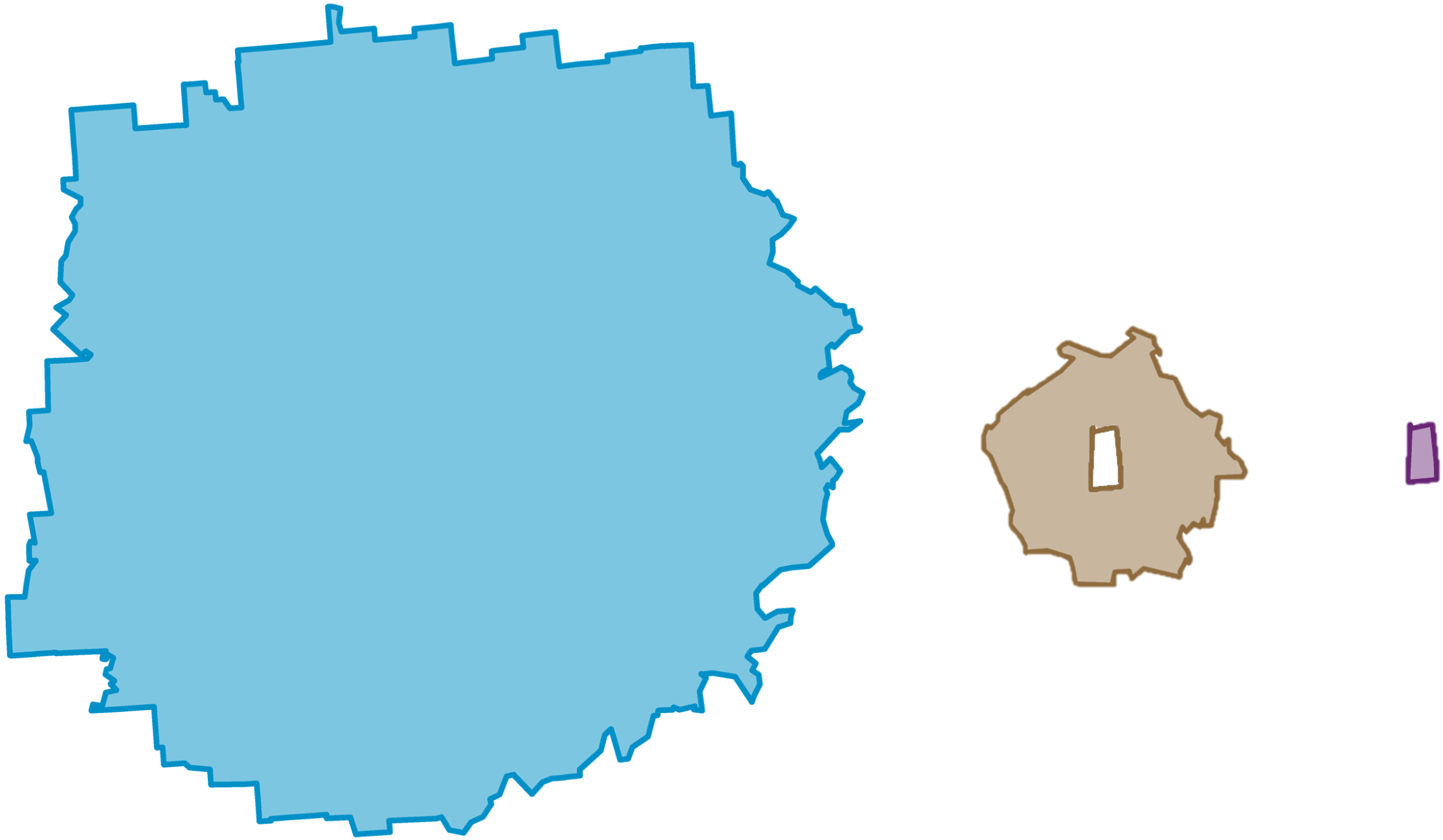

New Hot Spot
Consecutive Hot Spot
Intensifying Hot Spot
Persistent Hot Spot
Diminishing Hot Spot
Sporadic Hot Spot
Oscillating Hot Spot
Historical Hot Spot
New Cold Spot
Consecutive Cold Spot
Intensifying Cold Spot
Persistent Cold Spot
Diminishing Cold Spot
Sporadic Cold Spot
Oscillating Cold Spot
Historical Cold Spot
No trend detected

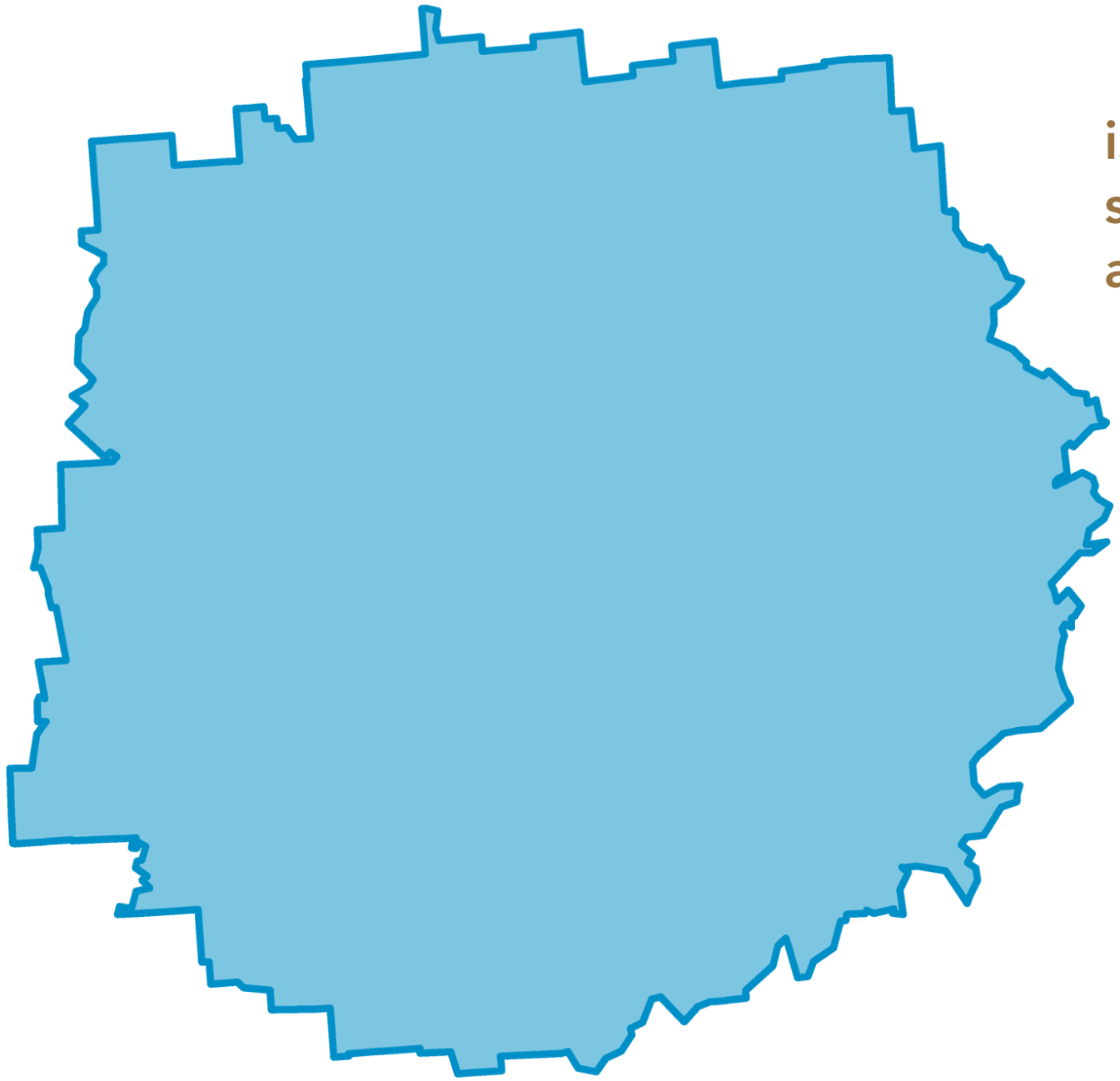
Cluster and Outlier Analysis

(Anselin Local Moran’s I)

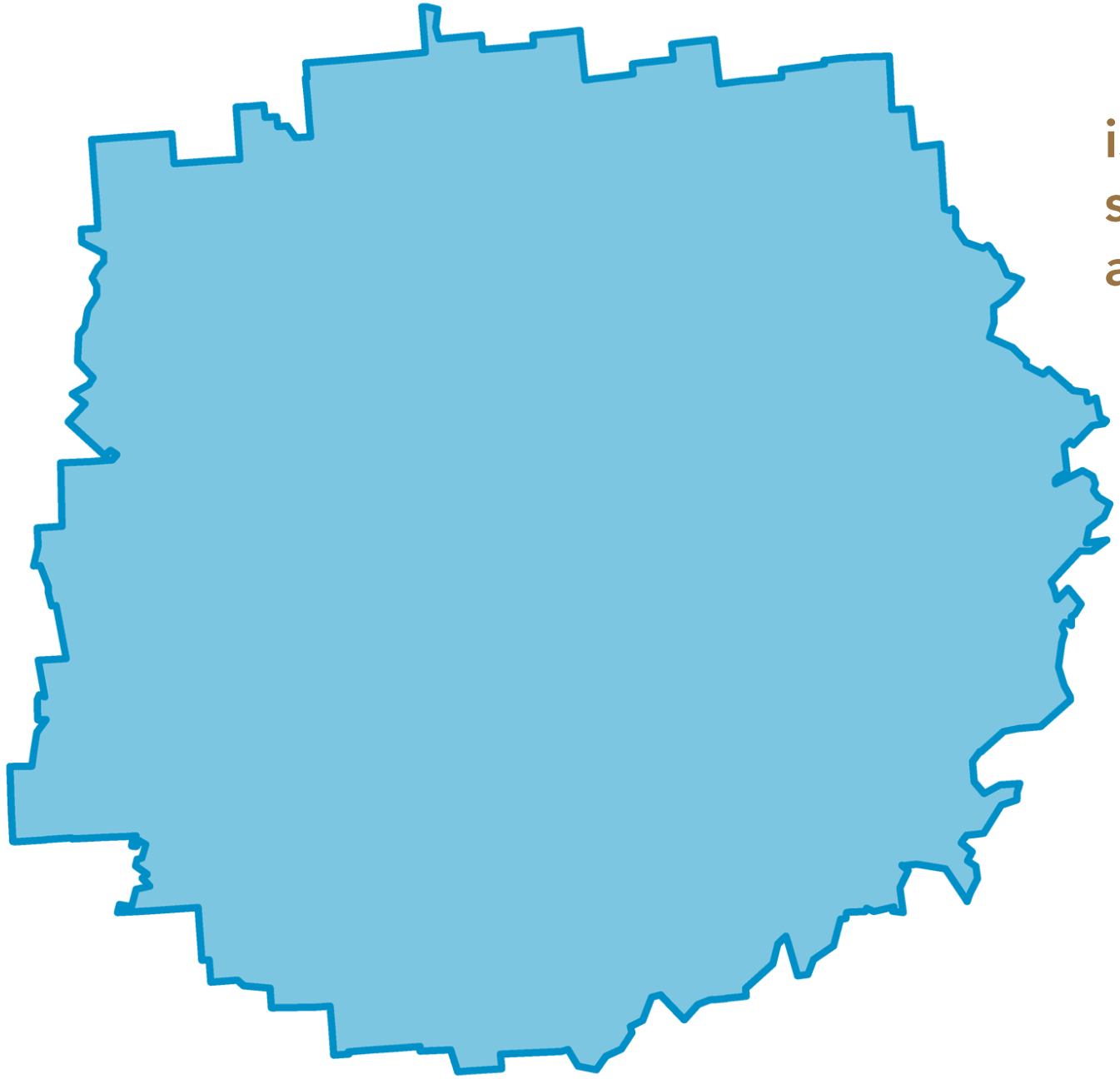
given a set of weighted features, identifies statistically significant hot spots, cold spots, and spatial outliers using the Anselin Local Moran's I statistic



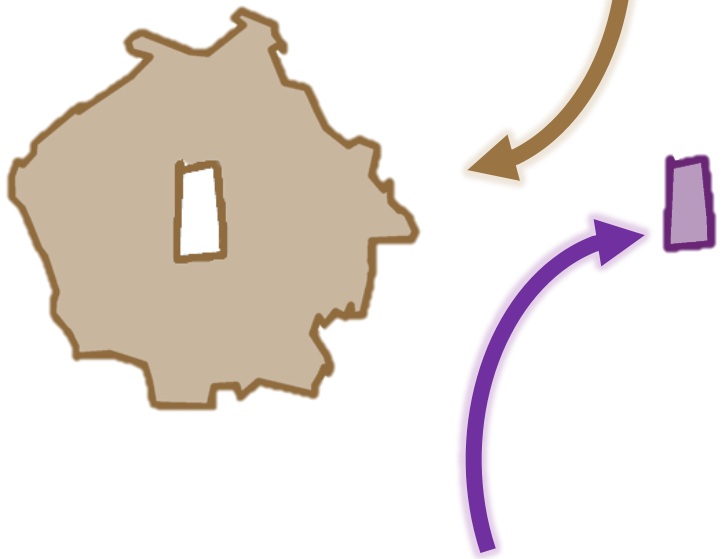




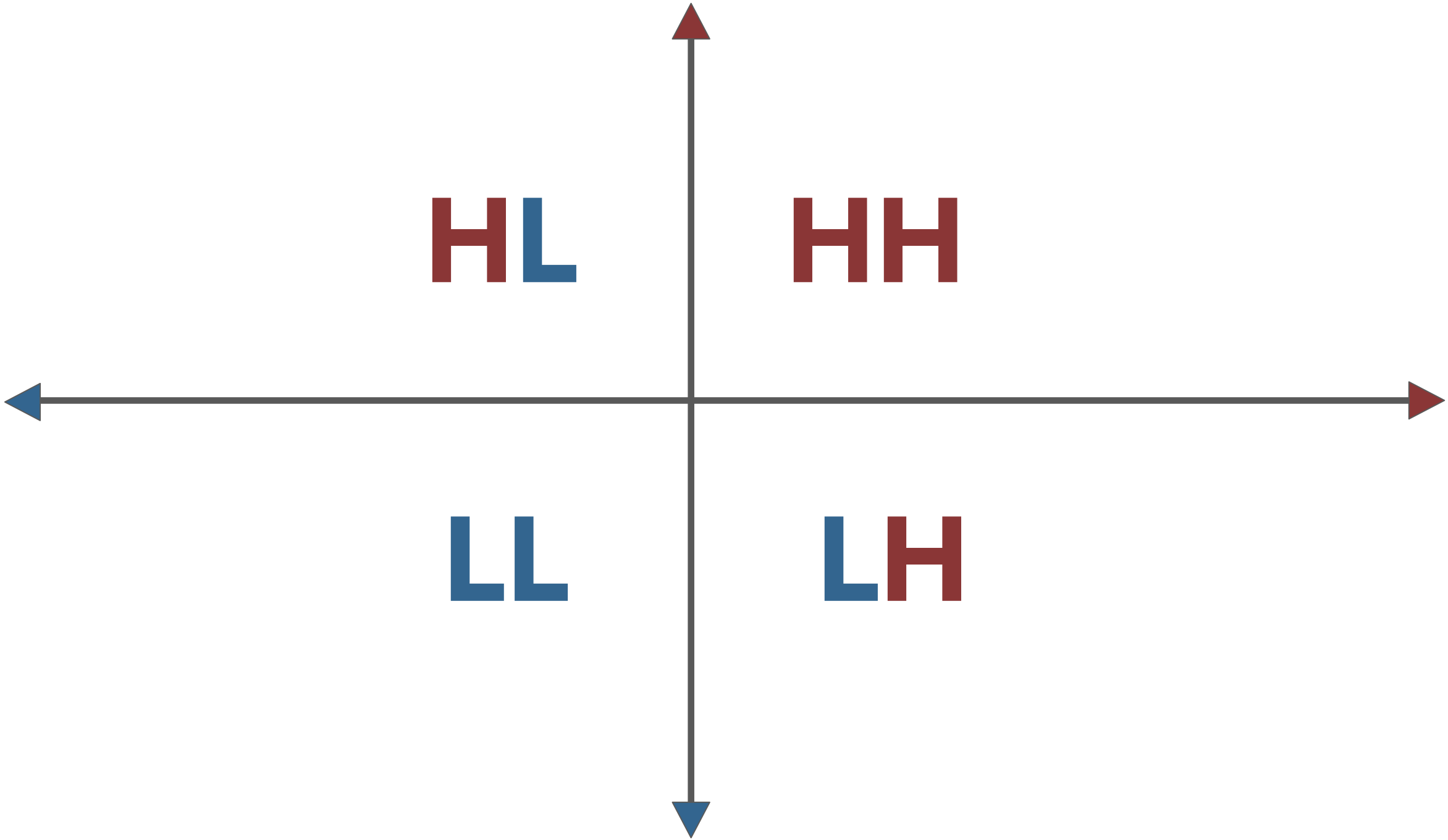
is the neighborhood
significantly different from
all other neighborhoods?

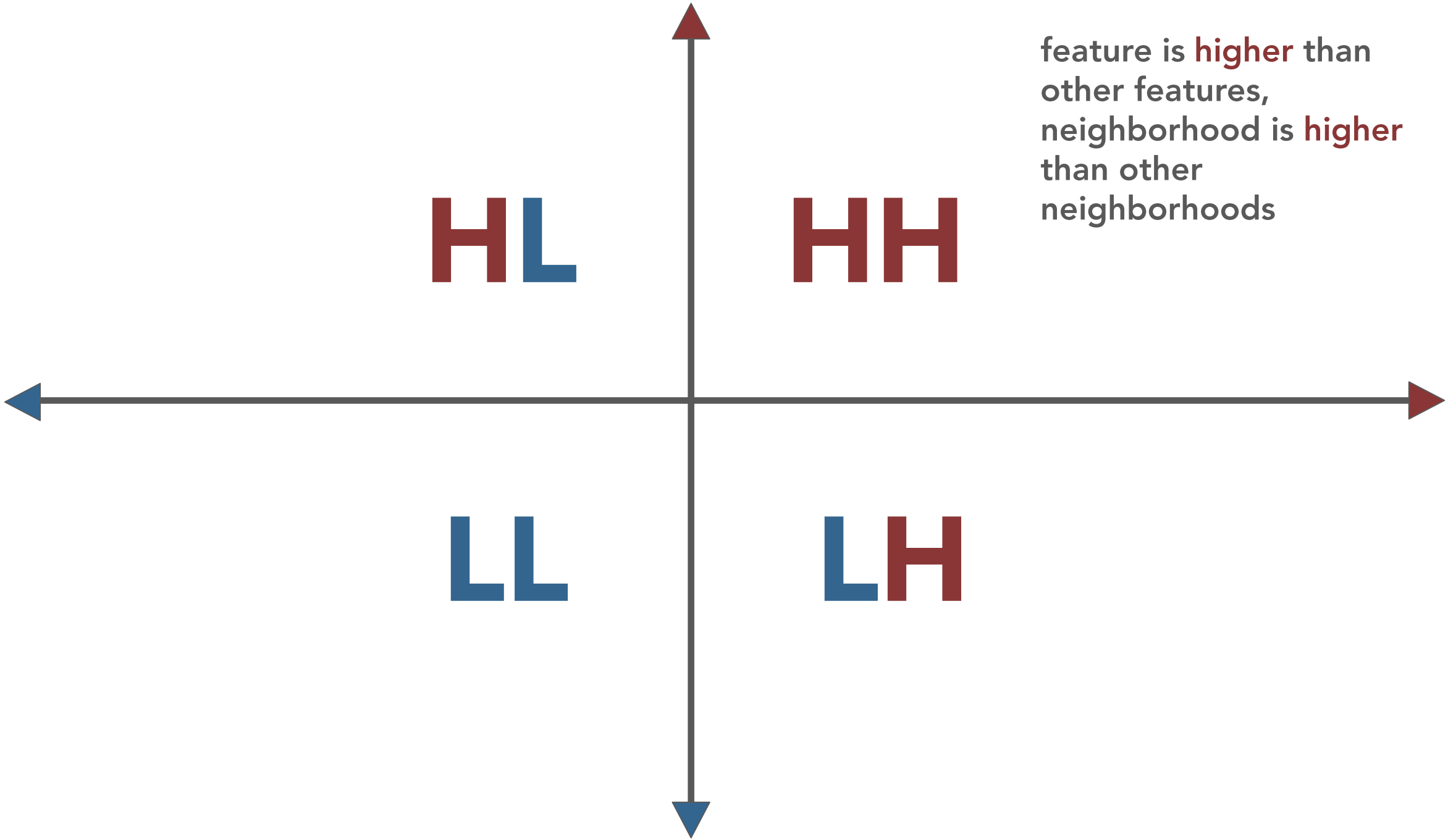


is the neighborhood significantly different from all other neighborhoods?

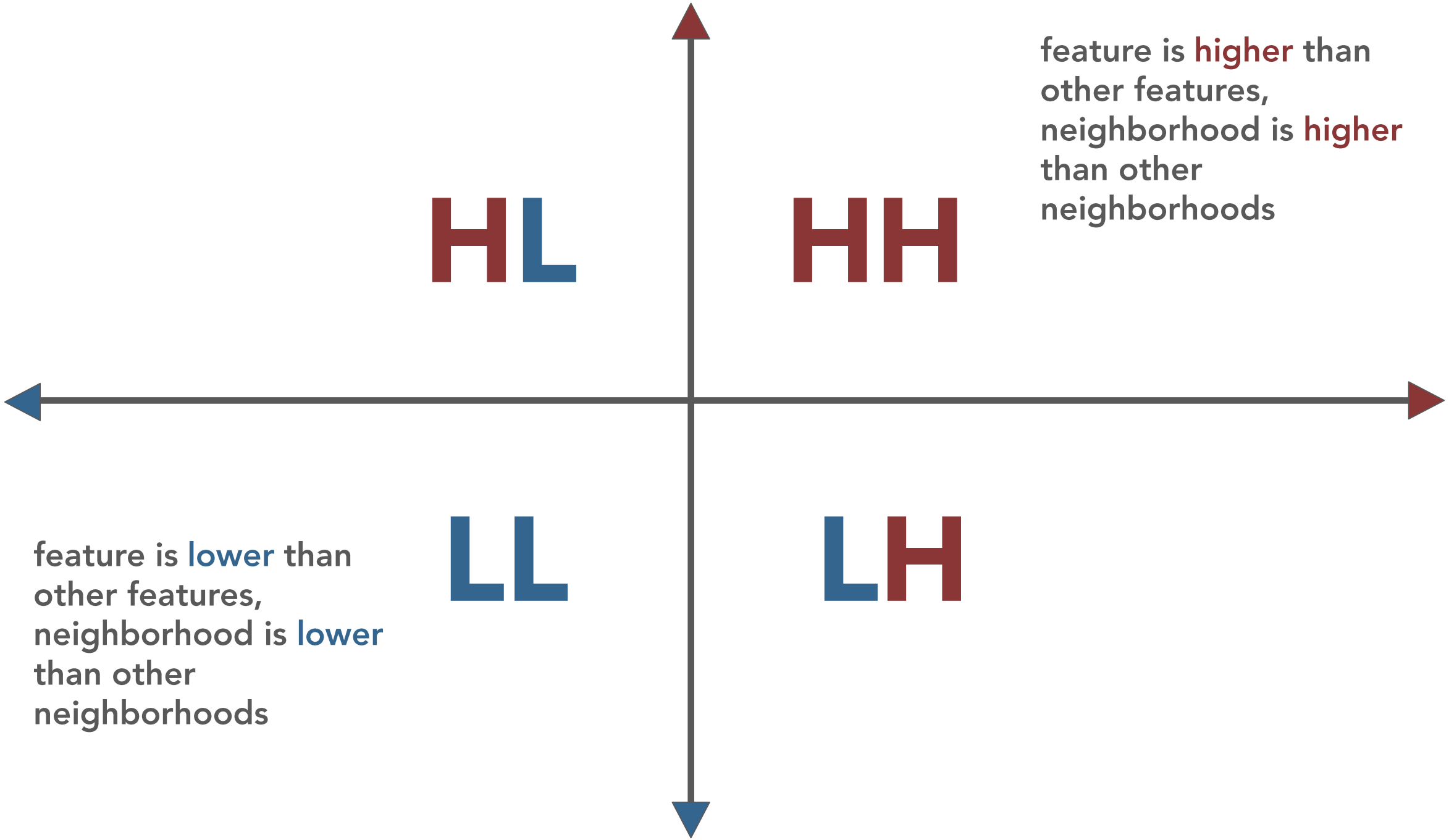


is the feature significantly different from all other features?


HL
HH
LL
LH

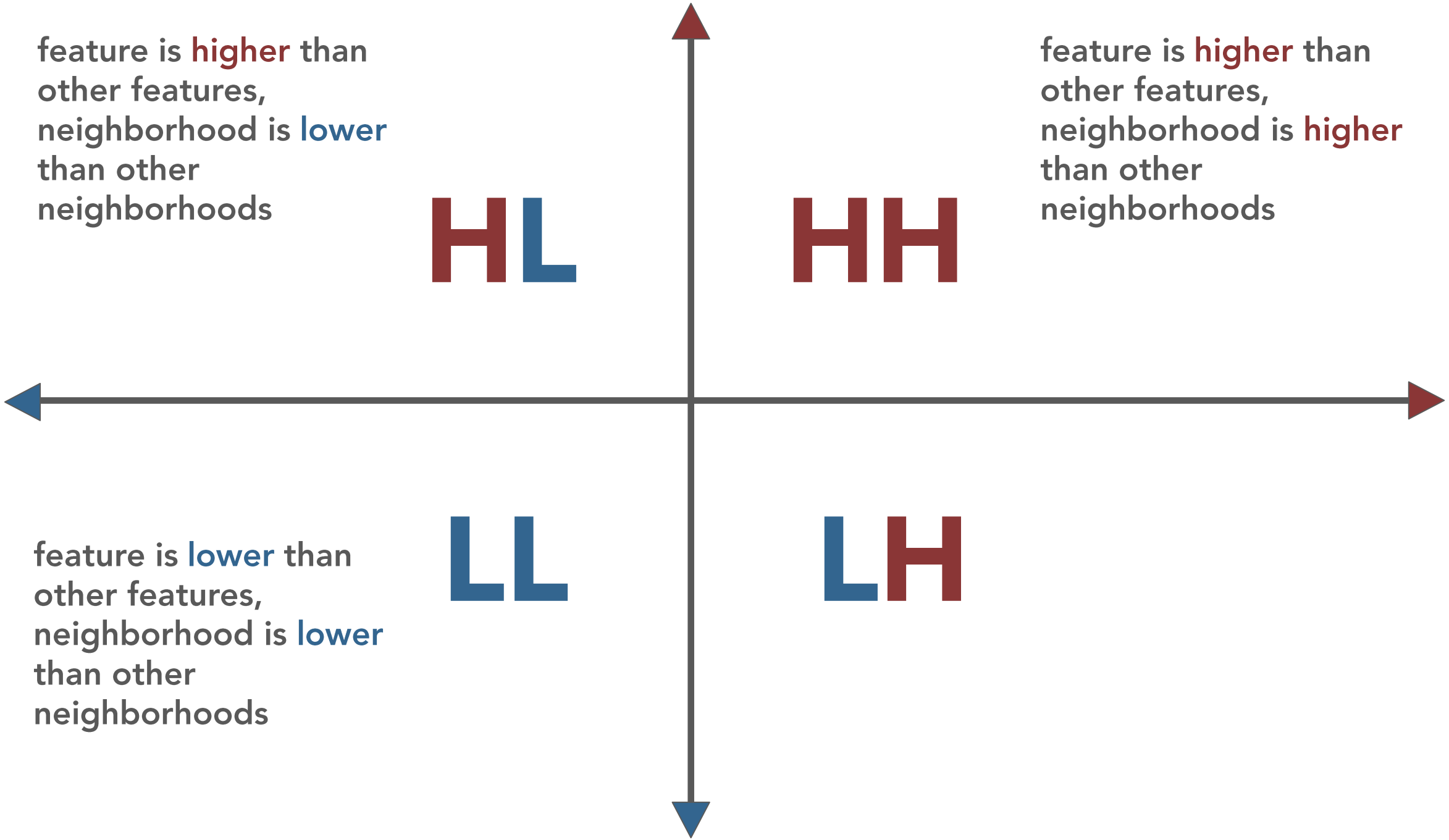


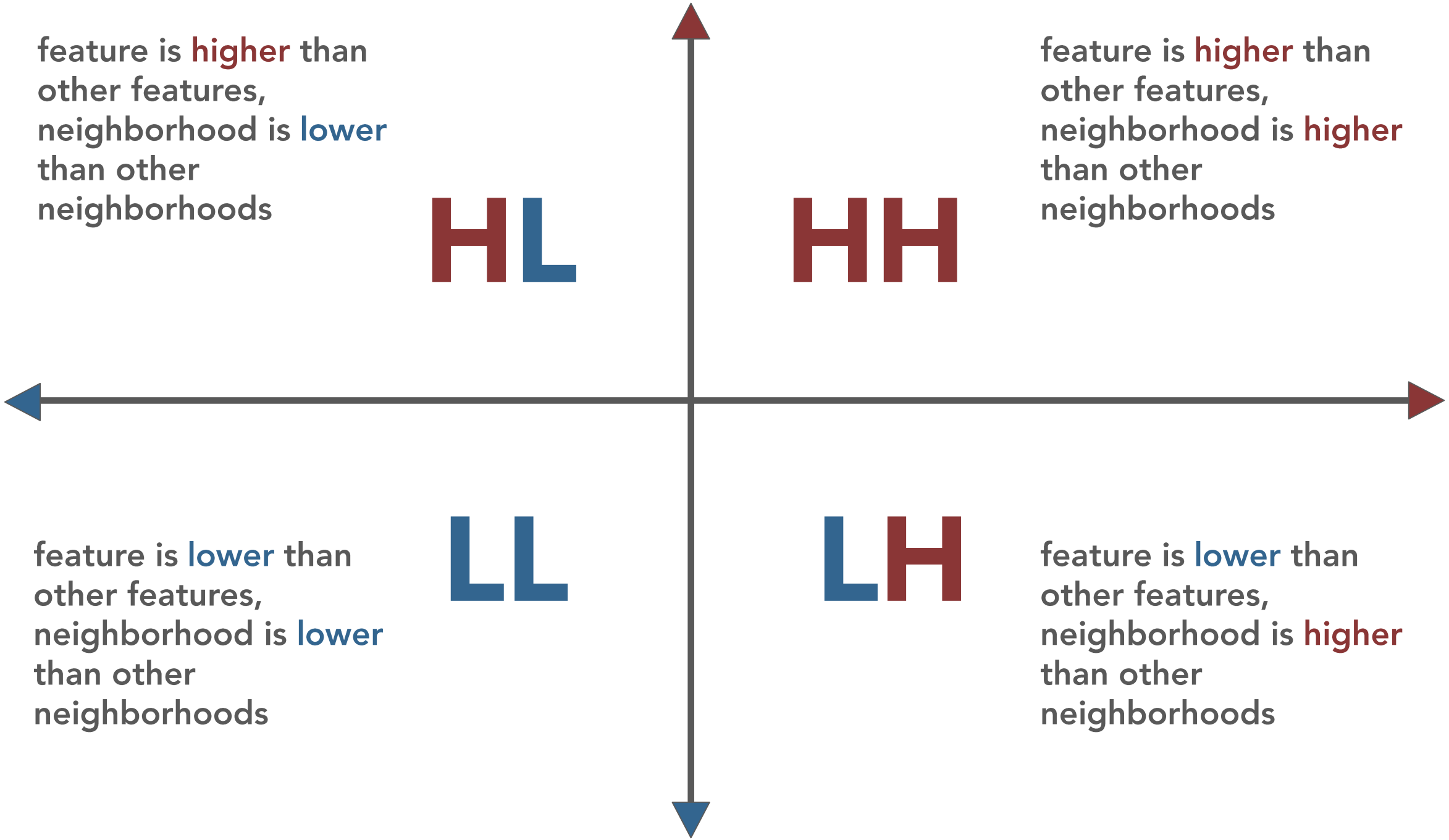
feature is higher than other features, neighborhood is higher than other neighborhoods



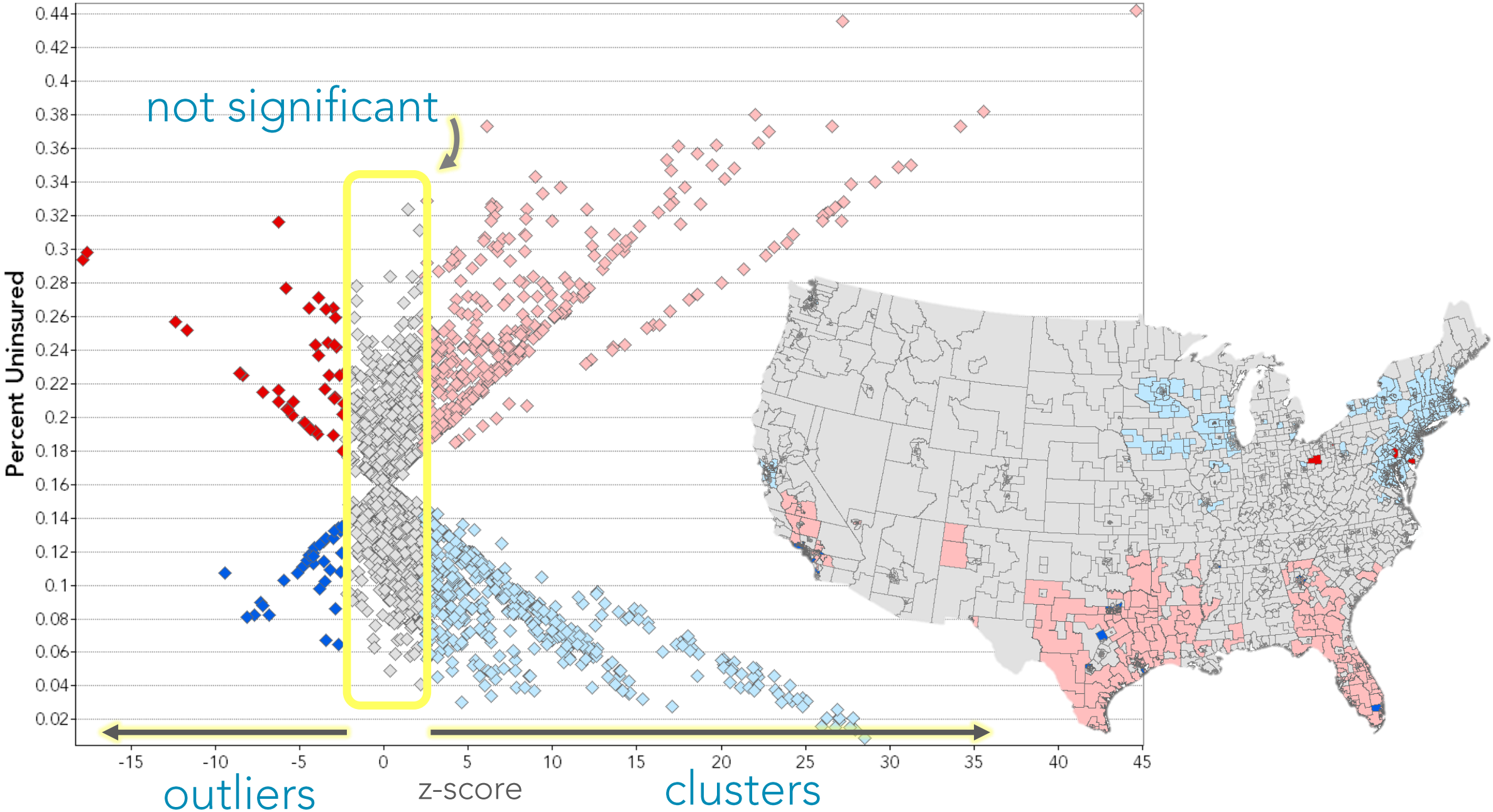
feature is higher than other features, neighborhood is higher than other neighborhoods

feature is lower than other features, neighborhood is lower than other neighborhoods

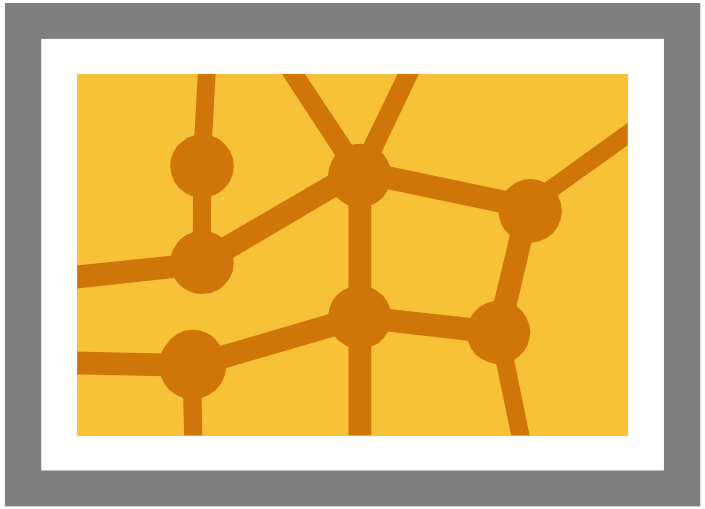

feature is higher than other features, neighborhood is lower than other neighborhoods
HL
HH
feature is higher than other features, neighborhood is higher than other neighborhoods
feature is lower than other features, neighborhood is lower than other neighborhoods
LL
LH

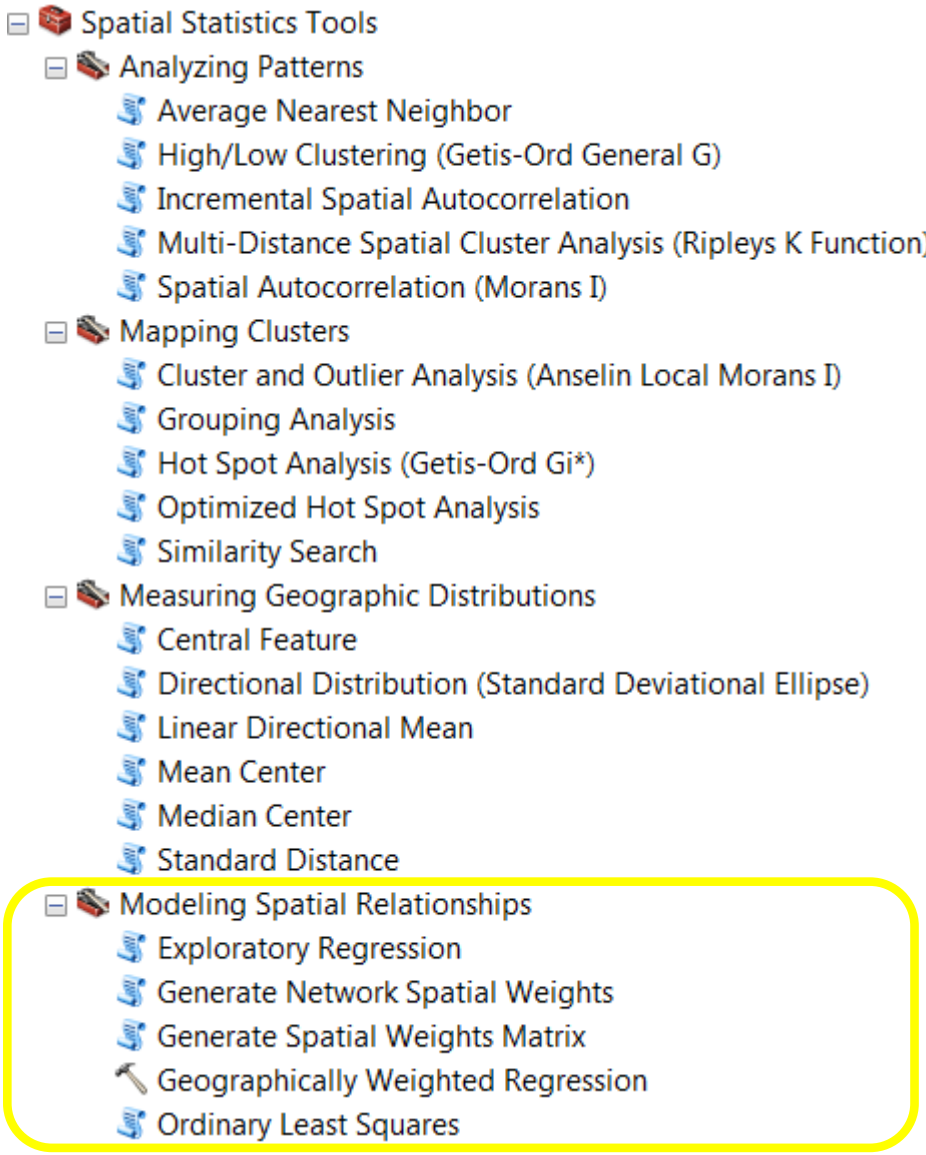

feature is higher than other features, neighborhood is lower than other neighborhoods
HL
HH
feature is higher than other features, neighborhood is higher than other neighborhoods
feature is lower than other features, neighborhood is lower than other neighborhoods
LL
LH
feature is lower than other features, neighborhood is higher than other neighborhoods

Cluster and Outlier Analysis



Modeling Spatial Relationships

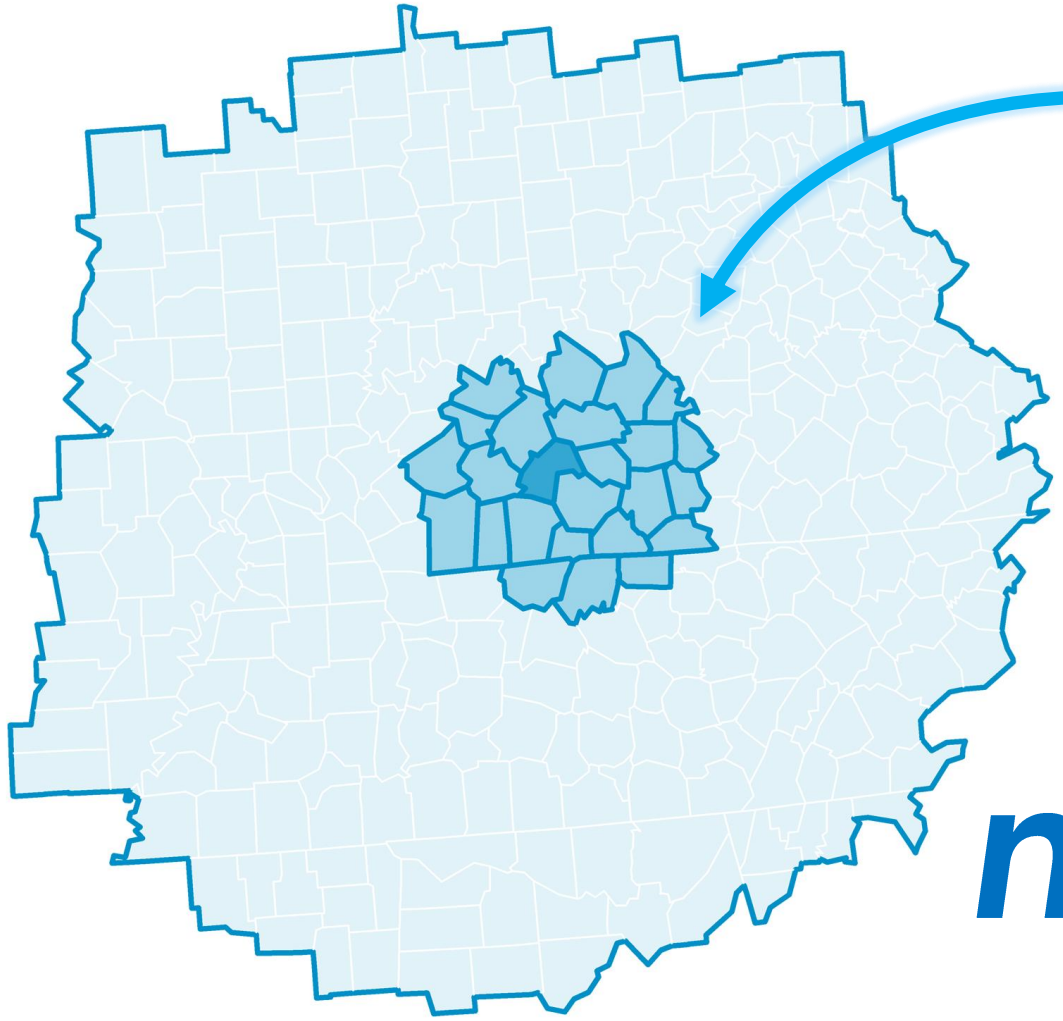




These tools model data relationships using regression analyses or construct spatial weights matrices.

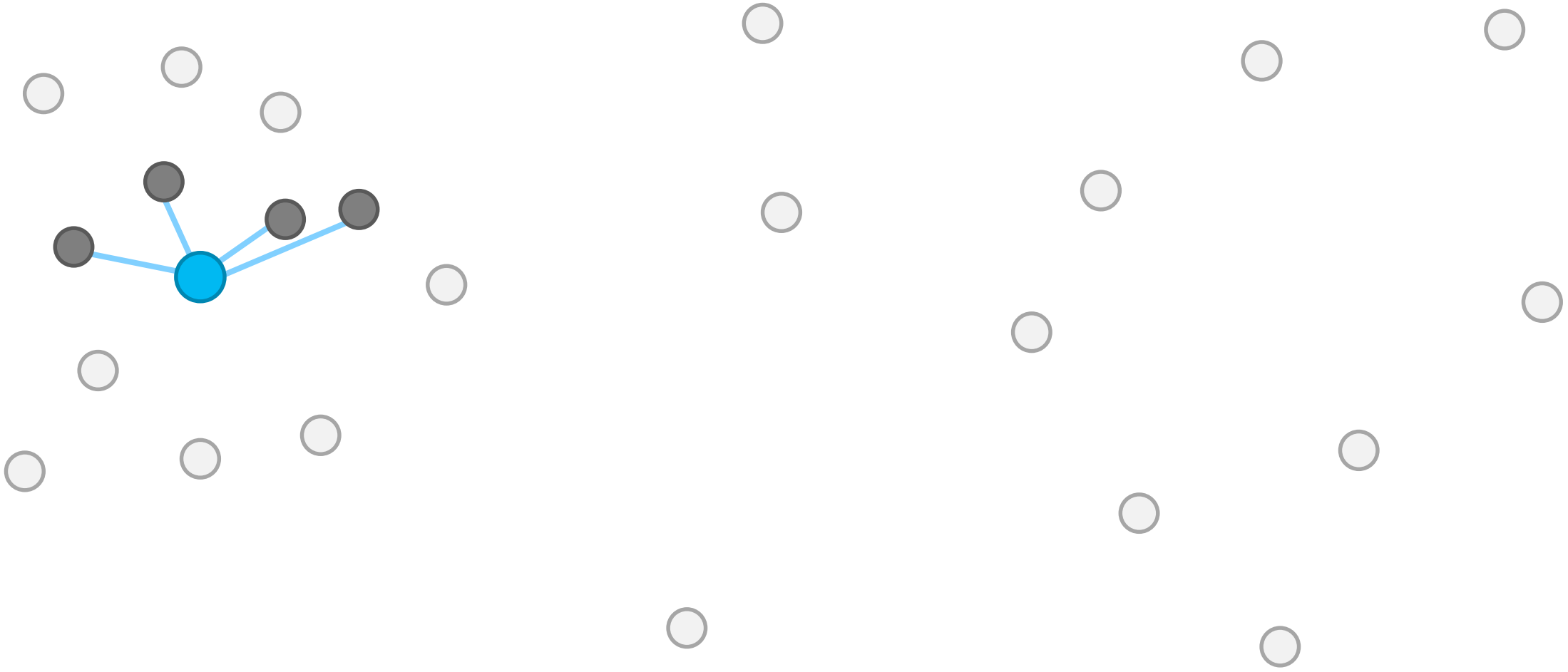
Generate Spatial Weights Matrix

constructs a spatial weights matrix (.swm) file to represent the spatial relationships among features in a dataset

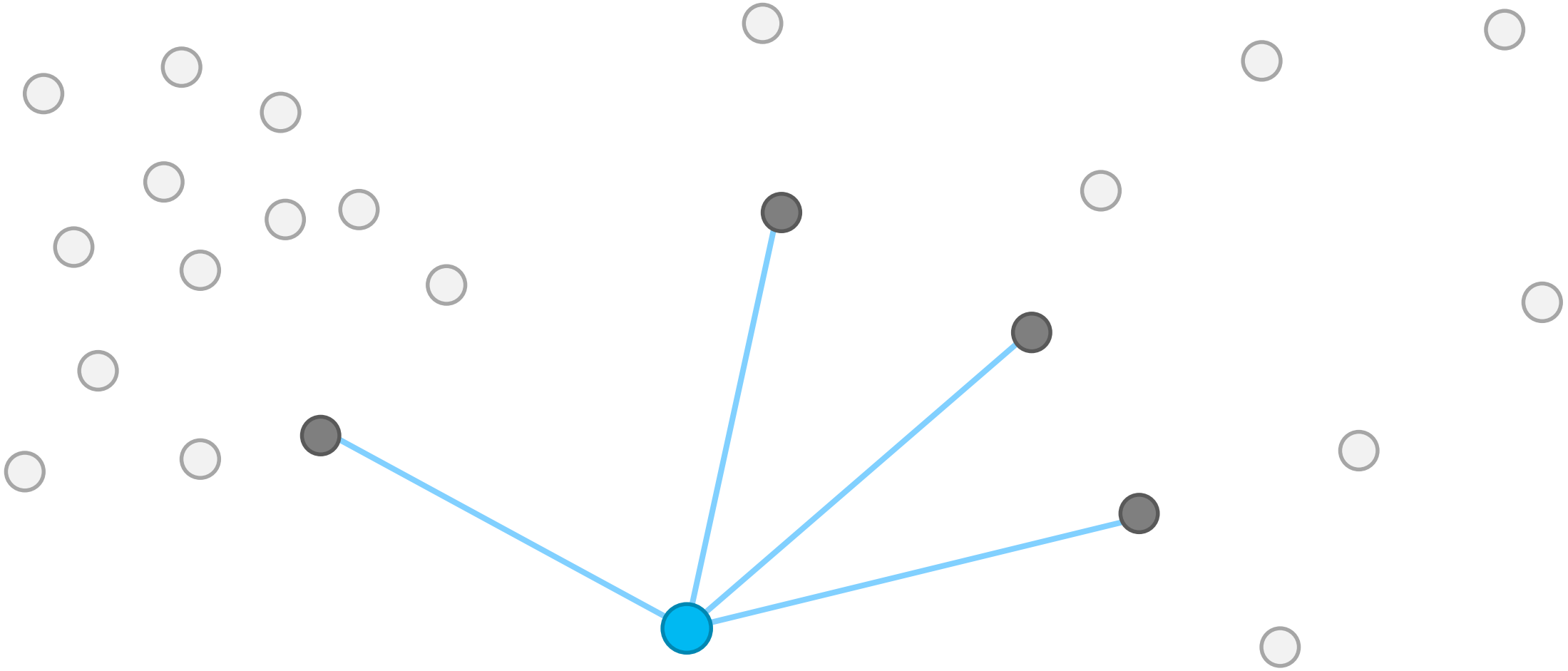


How are *neighborhood* sizes determined?

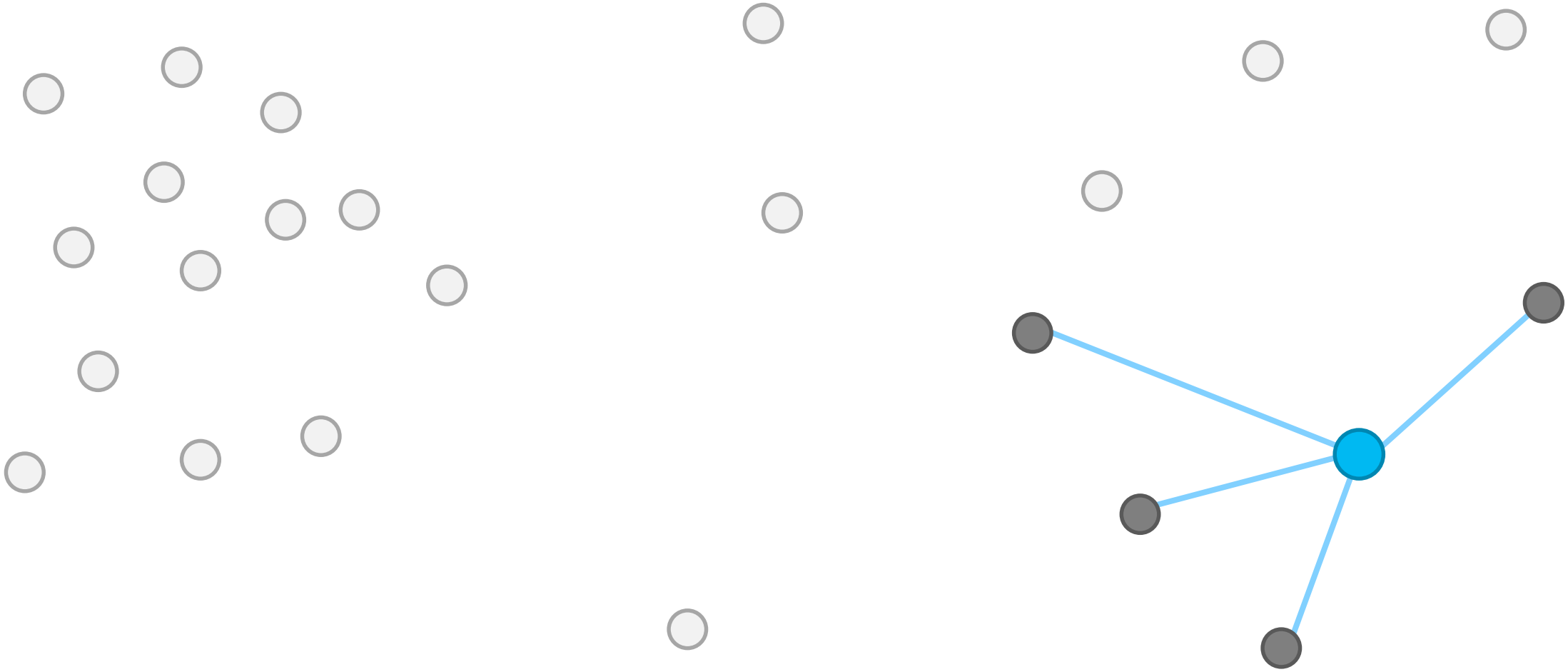
K Nearest Neighbors



K Nearest Neighbors

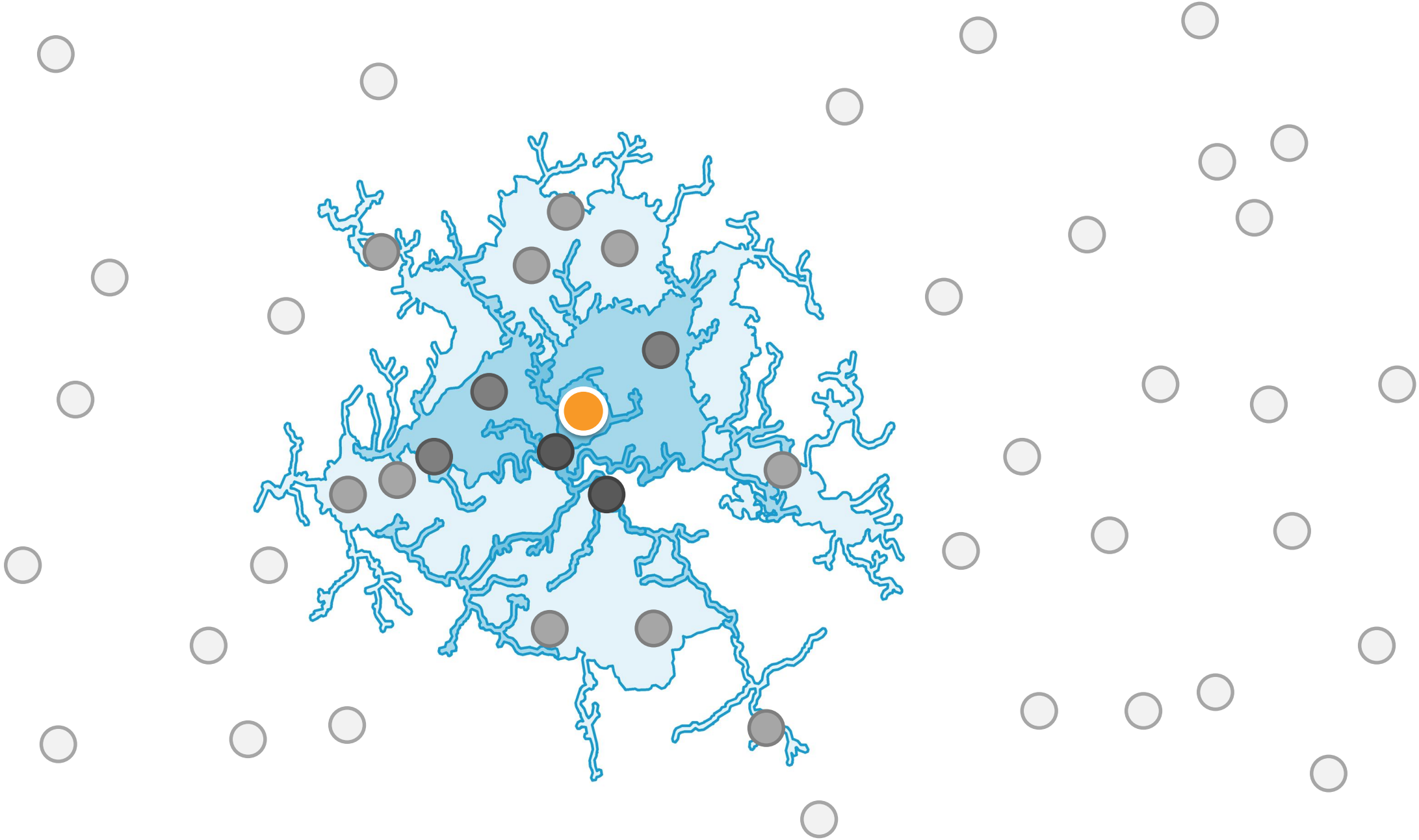


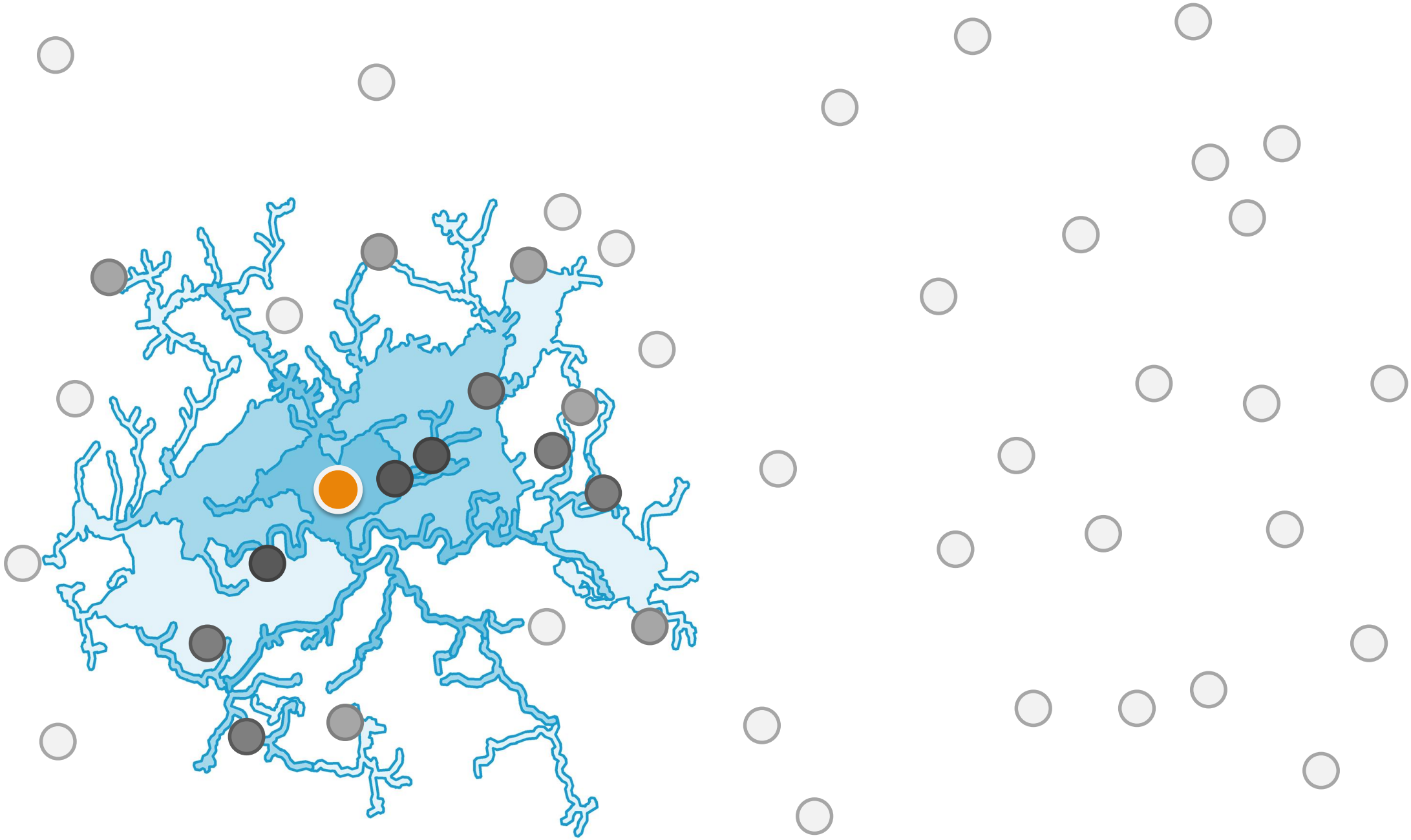
K Nearest Neighbors



Generate Network Spatial Weights

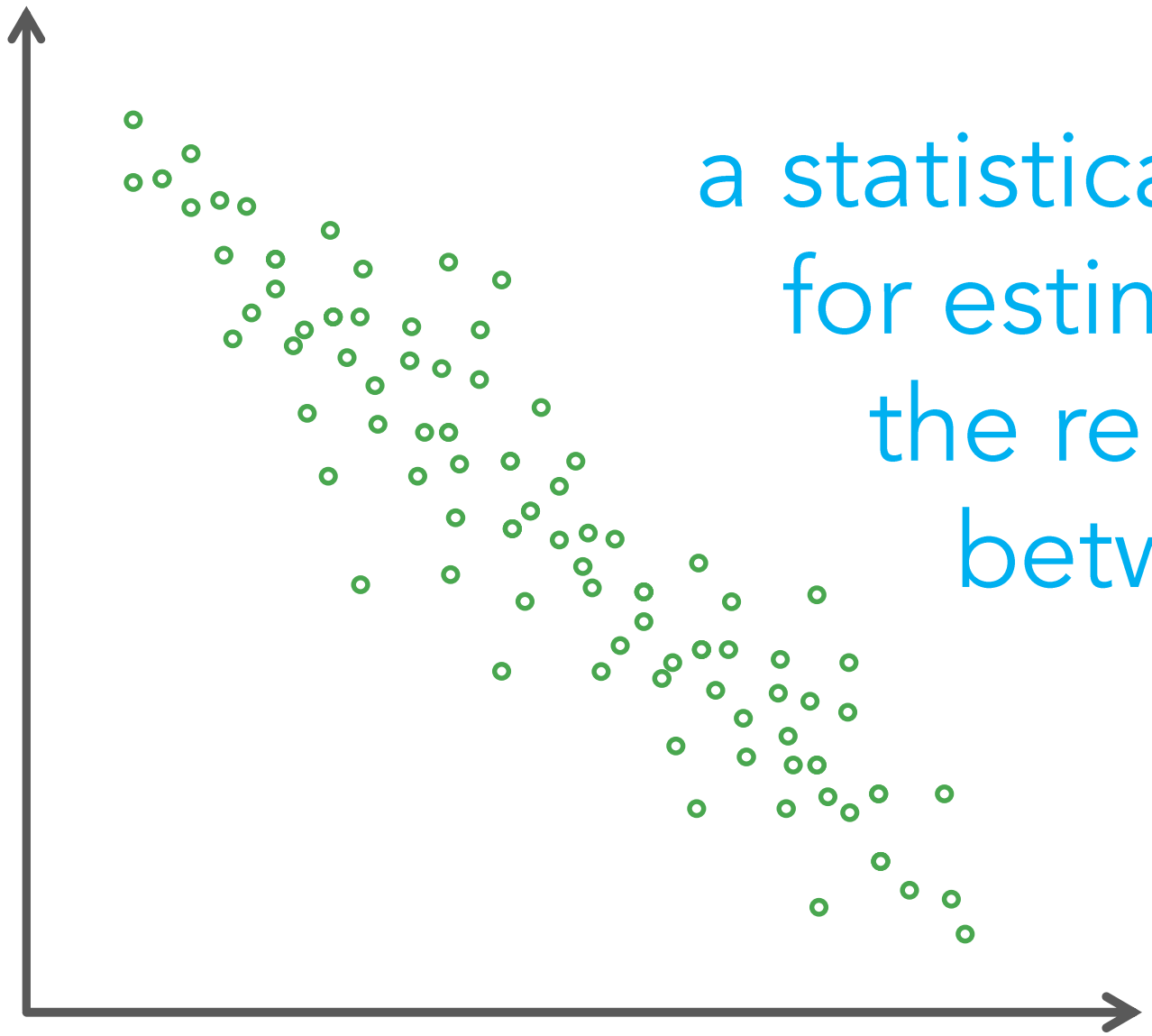
constructs a spatial weights matrix file (.swm) using a Network dataset, defining feature spatial relationships in terms of the underlying network structure





Ordinary Least Squares

performs global linear regression to model a dependent variable in terms of its relationships to a set of explanatory variables



a statistical process
for estimating
the relationships
between variables

Exploratory Regression

evaluates all possible combinations of the input candidate explanatory variables, looking for properly specified OLS models

Variables

Cultural

Environmental

Socio Economic

Lifestyle

Spatial

OLS

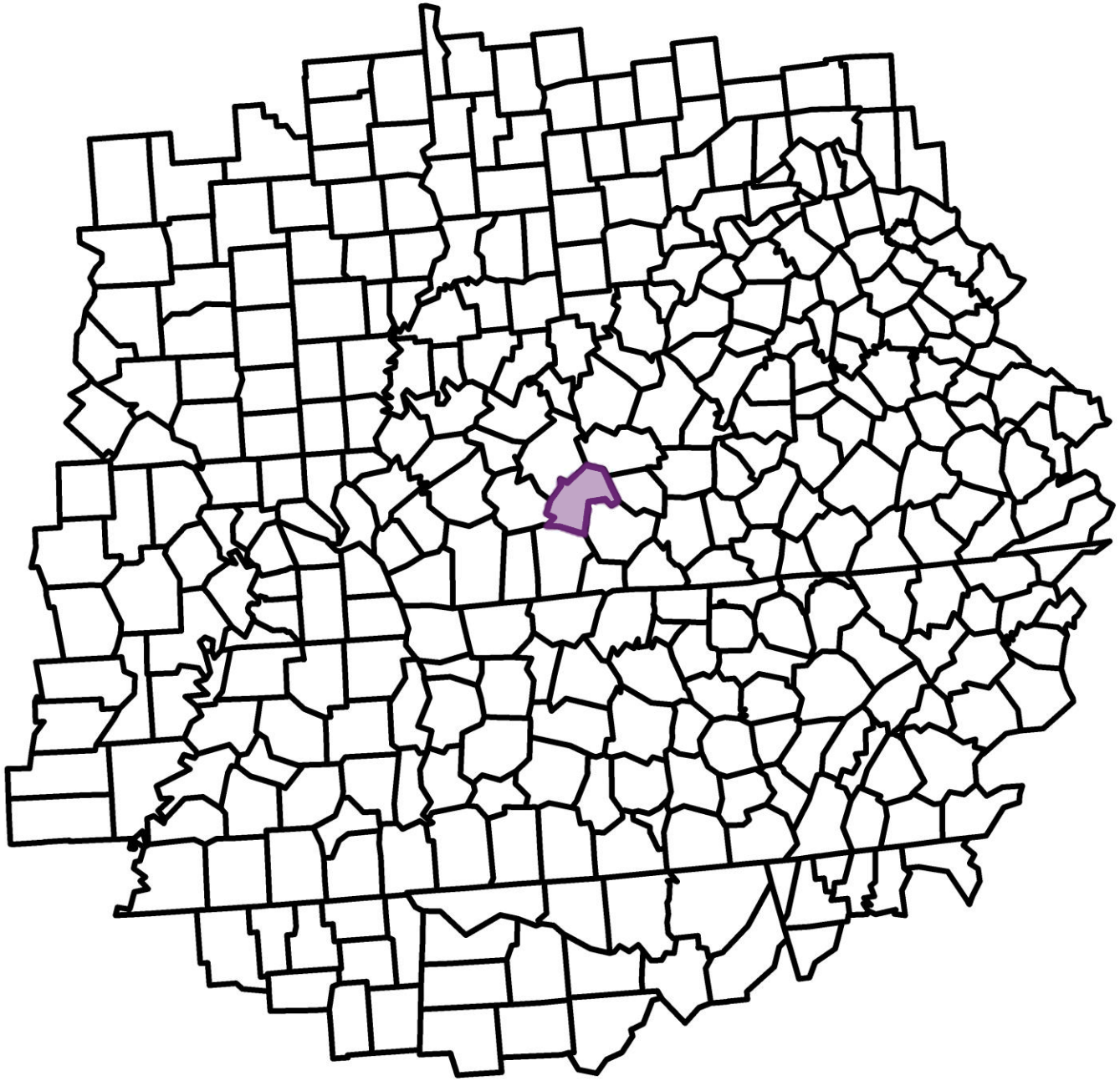
Tests all variable combinations for:

1. Redundancy
2. Completeness
3. Significance
4. Bias
5. Performance

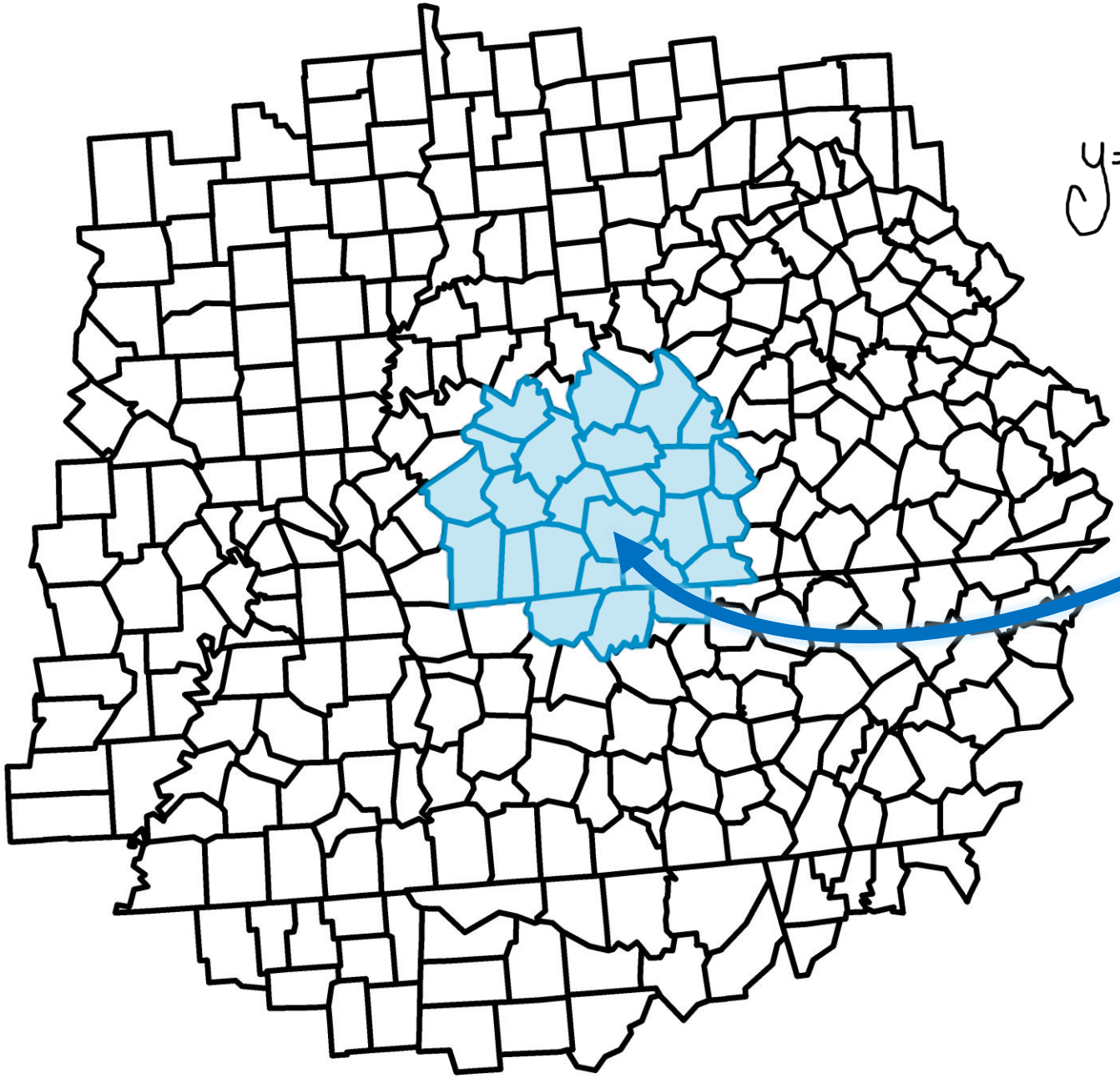
Creates Output Diagnostic Report

Geographically Weighted Regression

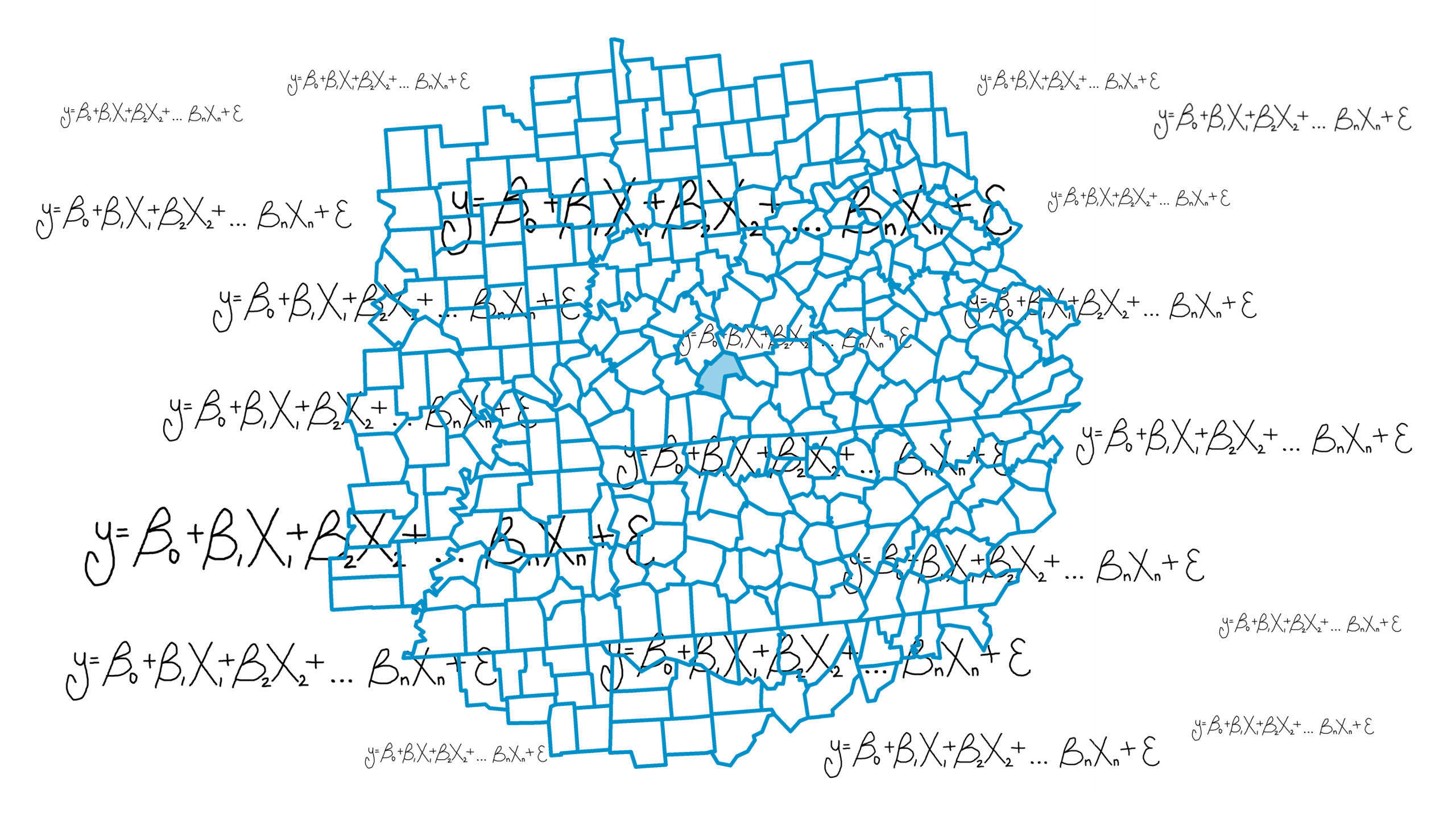
performs a local form of linear regression
used to model spatially varying relationships

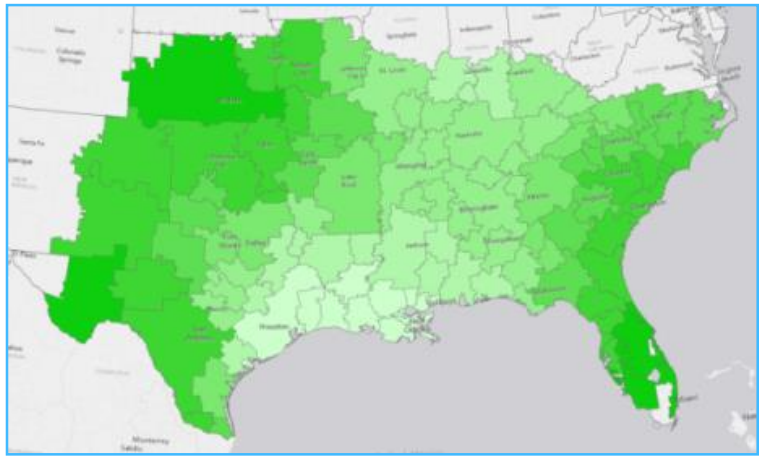


each feature gets a **separate equation**

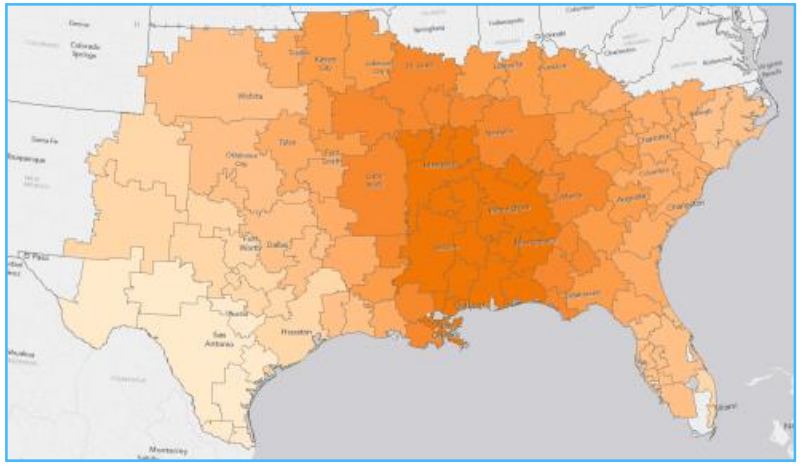


$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots \beta_n X_n + \varepsilon$$

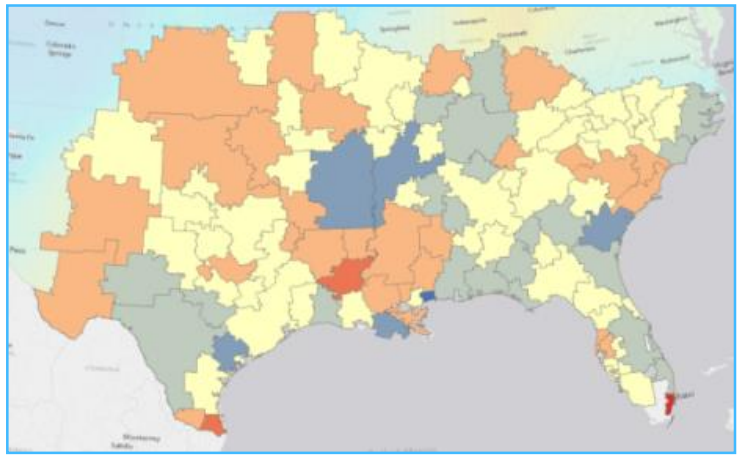

$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots \beta_n X_n + \varepsilon$



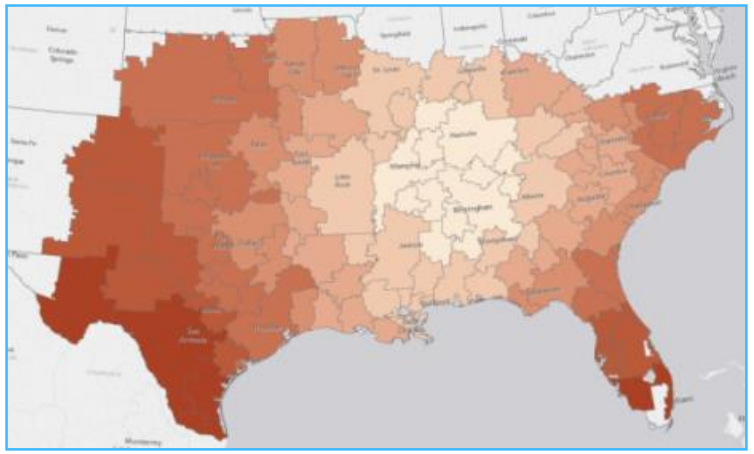
Local R-Squared



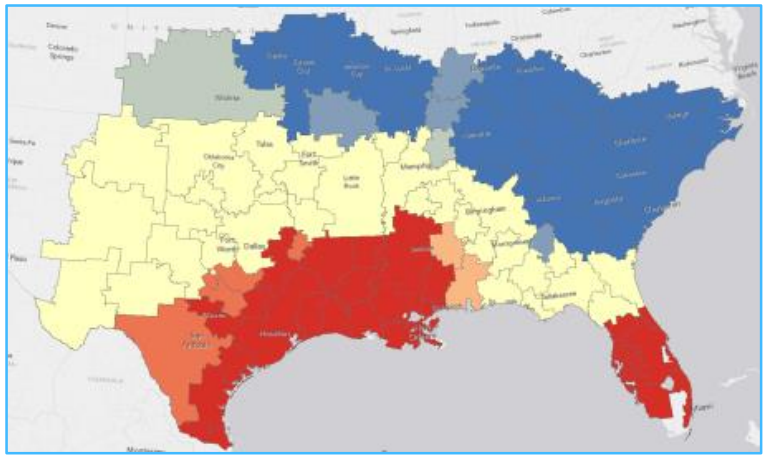
Coefficients



Residuals



Condition Number



Predictions

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